

VIRTUAL LSPs AT $e^+ e^-$ COLLIDERS

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ABSTRACT

Currently popular search strategies for supersymmetric particles may be significantly affected due to relatively light sneutrinos which decay dominantly into invisible channels. In certain cases the second lightest neutralino may also decay invisibly leading to two extra carriers of missing energy (in addition to the lightest supersymmetric particle (LSP)) – the virtual LSPs (VLSPs). It is shown that if the sneutrino masses happen to be in the small but experimentally allowed range ($m_{\tilde{\nu}} \approx 45\text{--}55$ GeV), these particles together with neutralino pairs may contribute significantly to the missing energy in the process $e^+e^- \rightarrow \gamma + \cancel{E}_T$ at LEP-2 energies as an enhancement over the Standard Model or the conventional MSSM predictions. It is further shown that a much larger region of the parameter space can be scanned at a high luminosity e^+e^- collider at 500 GeV like the proposed NLC machine. Moreover this process can play a complementary role to direct chargino searches at LEP-2 and NLC which may fail due to a near mass degeneracy of the chargino and the sneutrino. Formulae for the cross sections taking into account full mixings of the charginos and the neutralinos are derived. The signal remains observable even in the context of more restricted models based on $N=1$ SUGRA with common scalar and gaugino masses. The effect of soft photon brehmsstrahlung on the signal is also discussed briefly.

1 Introduction

It is well known that supersymmetry (SUSY) [1] is an attractive alternative to the Standard Model (SM) since it offers an elegant solution of the notorious naturalness problem, provided the masses of the superpartners are of the order of 1 TeV or less. The search for SUSY at the TeV scale is, therefore, a high-priority programme of current high energy physics. Extensive searches for SUSY at the present high energy accelerators including the Fermilab Tevatron and LEP-1 and LEP-1.5 have yielded negative results and have eliminated certain regions of the parameter space of the Minimal Supersymmetric extension of the Standard Model (MSSM)[1].

However, there are small but interesting regions of the parameter space, which are allowed by all experimental data, where the signatures of SUSY can be significantly different from the conventional ones considered in most cases. As an example, let us note that in most cases the search strategies for R -parity conserving SUSY particles are based on the assumption that there is a single, stable, weakly-interacting neutral superparticle, the so-called lightest supersymmetric particle(LSP). This particle, if produced, easily escapes detection and carries missing transverse energy (\cancel{E}_T). Moreover, as a result of R -parity conservation, all other superparticles decay into the LSP either directly or through cascades. Thus any sparticle production is accompanied by \cancel{E}_T , traditionally regarded as the most powerful weapon in the arsenal of SUSY hunters, carried by the LSP alone.

It has been emphasised in recent literature [2, 3] that in some interesting regions of the parameter space of the MSSM (with R -parity conservation) there could be other carriers of missing energy in addition to the LSP, due to sparticles which decay dominantly into invisible modes. In such a scenario the signals of sparticle production can be considerably different from the conventional ones. This happens in the following scenario.

The MSSM contains four spin- $\frac{1}{2}$ neutral particles. These particles are the superpartners of the photon, the Z -boson and the two neutral CP -even Higgs bosons. Linear combinations of these four states, the four neutral gauginos or neutralinos (\widetilde{N}_i , $i=1,4$), are the physical states. In the currently favoured models, the lightest neutralino(\widetilde{N}_1) is assumed to be the LSP [1]. Similarly, linear combinations of the superpartners of the W -boson and the charged Higgs boson give two physical charged gauginos or charginos.

The usual assumption that the MSSM is embedded into some Grand Unified Theory

(GUT) immediately implies, irrespective of the choice of any particular gauge group for the GUT, that the masses and the couplings of charginos and neutralinos depend only on three independent parameters. Usually these are taken as μ , $\tan\beta$ and the gluino mass $M_{\tilde{g}}$.

If no further assumption is made then the masses of the sfermions are totally independent of the gaugino-masses (we shall discuss below more restricted models with additional theoretical assumptions). Thus the sneutrinos ($\tilde{\nu}$, the superpartners of the neutrinos), though heavier than the LSP, could very well be lighter than the lighter chargino ($\tilde{\chi}_1^\pm$), the second lightest neutralino (\tilde{N}_2) and other superparticles. As a consequence, the invisible two-body decay mode $\tilde{\nu} \longrightarrow \nu \tilde{N}_1$ opens up and completely dominates over the others, being the only kinematically-allowed two-body decay channel for the sneutrinos. The other necessary condition for this scheme to work is that the \tilde{N}_1 has a substantial zino (superpartner of the Z -boson) component. This, however, is almost always the case as long as the gluino (\tilde{g} , the superpartner of the gluon) has a mass ($M_{\tilde{g}}$) in the range interesting for the SUSY searches at the Tevatron [4]. Moreover, in such cases the \tilde{N}_2 — which also has a dominant zino component — decays primarily through the process $\tilde{N}_2 \longrightarrow \nu \tilde{\nu}$. This, however, also requires the left and the right handed sleptons (\tilde{l}_L and \tilde{l}_R , the superpartners of leptons) to be heavier than \tilde{N}_2 . These two particles (\tilde{N}_2 and $\tilde{\nu}$), decaying primarily into invisible channels, may act as additional sources of \cancel{E}_T and can significantly affect the strategies for SUSY searches [2, 3]. They are, therefore, called *virtual* or *effective* LSPs (VLSPs or ELSPs) [2, 7] in the subsequent discussion.

Some consequences of the VLSP scenario (as opposed to the conventional MSSM where the LSP is the only source of missing E_T) in the context of SUSY search at both hadron and e^+e^- colliders have been discussed in the literature [2, 3, 5, 6, 7]. Here we wish to reiterate that for LEP experiments beyond the Z -pole the predictions of the VLSP scenario are significantly different from the conventional ones. For example, experiments at LEP-1.5 [8] have recently reported some improved bounds on the chargino-neutralino sector. These bounds are derived from the processes (a) $e^+e^- \longrightarrow \tilde{N}_1 \tilde{N}_2$ and (b) $e^+e^- \longrightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, assuming that $\tilde{\chi}_1^\pm$ and \tilde{N}_2 primarily decay into 3-body channels as predicted by the MSSM. In the VLSP scenario, however, the final state of process (a) is invisible. Thus the improved bounds on the neutralino sector from LEP-1.5 are not applicable in this scenario. Similarly in the presence of light $\tilde{\nu}$ -s, $\tilde{\chi}_1^\pm$ primarily decays (with branching ratio $\simeq 1$) into the hadronically quiet channel $l^\pm \tilde{\nu}$ [2, 5, 7]. Thus the bounds on the chargino sector derived from the absence

of events containing acoplanar jets and leptons and missing energy may have to be revised in this scenario. It will be interesting to use the absence of two acoplanar leptons in the above experiments to constrain the $(M_{\tilde{\chi}_1^\pm} - m_{\tilde{\nu}})$ mass plane in the VLSP scenario. However the constraints thus obtained will depend on $M_{\tilde{\chi}_1^\pm} - m_{\tilde{\nu}}$ and can be completely evaded if $\tilde{\nu}$ and $\tilde{\chi}_1^\pm$ are nearly degenerate so that the leptons in the final state are soft and unobservable. We now focus on a signal in the VLSP scenario which may lead to viable signals or constraints inspite of such a small mass splitting.

We consider the process $e^+e^- \longrightarrow \gamma + \text{nothing}(\not{E}_T)$, already discussed in a previous letter [6] in the context of **LEP-2**. Here we shall discuss the signal both at LEP-2 and at other future e^+e^- colliders at high energies. In the SM only $\nu\bar{\nu}$ pairs contribute to the final state. In the conventional MSSM both $\nu\bar{\nu}$ and $\tilde{N}_1\tilde{N}_1$ pairs contribute to this kind of effect. With VLSPs, however, there will be additional contributions from $\tilde{\nu}\tilde{\bar{\nu}}$ and $\tilde{N}_i\tilde{N}_j$ ($i, j = 1, 2$) which tend to increase the cross section quite significantly. In [6] it was found that a significant enhancement of the cross section over the prediction of the SM occurs at LEP-2 in a reasonable region of the MSSM parameter space (see section 2 for the details) allowed by the experimental data (most notably from LEP-1 [9]). Moreover, the bulk of the extra contribution comes from $\tilde{\nu}\tilde{\bar{\nu}}$ pairs. Thus, such a signal, if detected, can be distinguished not only from the SM but also from the conventional MSSM without VLSPs.

In this work we have elaborated the results of [6] with further details. The scan over the LEP-1 allowed parameter space is now more complete. This, however, does not alter the results of [6] qualitatively, although some quantitative changes are noted. Assuming a conservative detector design as in [6] we have found that at LEP-2 the statistical significance of the signal is rather modest. For optimistic choices of SUSY parameters (most notably for relatively low sneutrino and gluino masses, $m_{\tilde{\nu}} = 45\text{--}60$ GeV, $M_{\tilde{g}} \simeq 200\text{GeV}$), signals with statistical significance $\geq 3\sigma$ can be obtained (numerical details are given in the next section).

The cross sections for the process $e^+e^- \longrightarrow \gamma + \text{nothing}(\not{E}_T)$ has been discussed extensively in the literature. We have done a complete calculation in the VLSP scenario without using the simplifying assumptions used in earlier works. In [6] we presented some of the numerical results. But the formulae for the cross sections, which are quite cumbersome, could not be presented in a brief letter. A major result of this paper is the detailed formulae presented in a compact form. First we have calculated the full cross section for the purely SM process $e^+e^- \longrightarrow \gamma\nu\bar{\nu}$. In many of the earlier works [10], appropriate for LEP-1, the

contribution of the W -exchange diagrams was computed in the limit of four-fermion contact interaction. We have recalculated it with the full W -propagator. We have also taken the widths of W and Z into account. Our results agree completely with those of [10] after taking the appropriate limits. In [11] this cross section was also computed without any approximation. However, the published results include several misprints (see, for example, equation(3) which contains several terms which are dimensionally incorrect). This makes comparison rather difficult. This cross section was also computed in [12] by neglecting the widths but keeping the full W -propagator. Their analytical formulae agree completely with ours in the appropriate limit. Moreover, a comparison of the numerical results shows that effects of the widths are indeed negligible, at least for the energy ranges considered in this paper.

The most important contribution to this process in the VLSP scenario comes from $e^+e^- \longrightarrow \gamma\tilde{\nu}\tilde{\nu}$. Only the amplitudes of the relevant Feynman diagrams are given in, for example, [13] in the limit when the chargino is purely a wino (superpartner of the W -boson). We have computed the full cross section taking into account the chargino-mixing matrix. Our numerical results agree with those of [13] in the appropriate limit. In a more recent paper [18] this cross section has been computed by assuming the charginos to be very massive. In this limit we agree with the main features of their results.

We have also computed the cross section for the process $e^+e^- \longrightarrow \gamma\tilde{N}_i\tilde{N}_j$, ($i, j = 1, 2$) taking the 4×4 neutralino mass matrix into account. This cross section with only LSP-pairs ($i = j = 1$) in the final state is also relevant for the conventional MSSM and was computed in [14] in the limit when the \tilde{N}_1 is a pure photino without any mixing. In this approximation the s-channel Z -exchange diagrams are absent which reduces the number of diagrams and interferences between them. Our numerical results agree, in the appropriate limit, with the those of [14]. In a very recent paper [15] the calculation for a mixed LSP has been done using the structure function approach [16]. One of the conclusions of [6], *viz.* LSP pairs alone cannot give a signal with acceptable statistical significance, is supported by [15]. The general formulae presented in this paper also include the contribution of $\tilde{N}_1\tilde{N}_2$ and $\tilde{N}_2\tilde{N}_2$ pairs.

In Appendix-A we present analytical formulae for all the matrix elements squared.

Using these results we have also computed the cross sections at e^+e^- colliders at higher energies after introducing kinematical cuts to reduce the SM backgrounds. Many of these machines are likely to be of very high luminosities[17]. As a consequence of this, signals

of very high statistical significance ($> 5\sigma$) can be obtained at CM energies ≈ 350 and 500 GeV which are attainable at the proposed Next Linear Collider(NLC). Special care, however, should be taken to reduce the background from radiative Bhabha scattering where both the final state charged particles are lost in the beam pipe [18]. This will be discussed in further details in section 3.

The VLSP scenario, which is certainly consistent with all available experimental results on SUSY searches, can also be accommodated in the more constrained and theoretically motivated models based on $N = 1$ Supergravity with common scalar and gaugino masses at a high scale [19]. In this scenario the sneutrino and the gaugino masses are not completely independent, but get related through the renormalisation group (RG) equations. It was shown in [7] that the VLSP scenario can be accommodated even in this highly constrained scenario. In this paper we have found that the signal at LEP-2 is reasonable for certain regions of the parameter space given in [7], while at NLC, signals with high statistical significance can still be obtained..

It may be noted at this point that in [6] only the lowest order cross section was considered. In this paper we have considered the effects of soft photon brehmstrahlung on the cross section. These effects can be obtained to all orders in perturbation theory by using, e.g., the structure function approach of [16]. We have found that the impact of these corrections on the signal is rather modest for the entire energy range considered by us and it leaves the signal to (root)background ratio almost unaffected.

The plan of the paper is as follows. In section 2 we consider the signal at $\sqrt{s}=190$ GeV corresponding to the LEP-2 energies and briefly comment on the possibilities at LEP-1.5 In section 3 the same discussion is carried out for high luminosity e^+e^- colliders operating at higher energies. In section 4 the signal is discussed in the context of highly constrained models based on $N = 1$ SUGRA. Section 5 discusses briefly the effect of soft photon brehmsstrahlung on the signal. Our conclusions are summarised in section 6. The relevant formulae for the cross sections are given in the Appendix.

2 The Signal at LEP-2 Energies

In our calculations we use the usual assumption of a common gaugino mass at the GUT scale which relates the U(1) gaugino mass M_1 with the SU(2) gaugino mass M_2 [1]. With this

assumption M_2 , the higgsino mass parameter μ and $\tan \beta$ determines the masses and the couplings of the charginos and neutralinos completely. In some of our figures we have used $3M_2$ as the variable instead of M_2 . The advantage is that this is approximately equal to the running gluino mass (through the above assumption of unification) which can be directly related to the constraints obtained from the Tevatron [4]. We, however, emphasise that this equality is only approximate and the factor 3 may change, though not drastically, with the energy scale. Nevertheless we shall denote in the following $3M_2$ by the gluino mass ($M_{\tilde{g}}$) for the sake of simplicity.

In addition we have assumed the $SU(2)$ breaking relation :

$$m_{\tilde{e}_L} = \sqrt{m_{\tilde{\nu}}^2 + \cos^2\theta_W D_Z}$$

where

$$D_Z = M_Z^2 \frac{\tan^2\beta - 1}{\tan^2\beta + 1}$$

and $m_{\tilde{\nu}}$ is treated as a free parameter and three degenerate sneutrinos are assumed. For the right handed sleptons we have made the popular assumption $m_{\tilde{e}_R} \approx m_{\tilde{e}_L}$ although deviations from this approximation may naturally occur in some models.

In the VLSP scenario the following constraints must be satisfied [7]:

$$\begin{aligned} m_{\tilde{\nu}} &< M_{\tilde{N}_2} < m_{\tilde{e}_L}, m_{\tilde{e}_R} \\ m_{\tilde{\nu}} &< M_{\tilde{\chi}_1^\pm} < m_{\tilde{e}_L} \end{aligned}$$

In Figs.[1A] and [1B] we present the regions in the $(M_{\tilde{g}} - m_{\tilde{\nu}})$ mass plane (where the precise definition of the parameter $M_{\tilde{g}}$ is given in the first paragraph) compatible with the above inequalities for $\mu = -250$ (Fig.1A) or $+250$ (Fig.1B) GeV, $\tan\beta=10$, $150 \text{ GeV} \leq M_{\tilde{g}} \leq 800 \text{ GeV}$ and $m_{\tilde{e}_L} = m_{\tilde{e}_R}$. In each figure the entire bounded area corresponds to the region of the parameter space where $\tilde{\nu}$ -s behave like VLSP-s. Corresponding to each $m_{\tilde{\nu}}$, this happens for a range of $M_{\tilde{g}}$. The lower limit of this range comes from the condition $m_{\tilde{\nu}} < M_{\tilde{N}_2}, M_{\tilde{\chi}_1^\pm}$ while the upper limit comes from $M_{\tilde{N}_1} < m_{\tilde{\nu}}$. If the additional condition $M_{\tilde{N}_2} < m_{\tilde{e}_L}, m_{\tilde{e}_R}$ is satisfied then \tilde{N}_2 also decays invisibly. This happens in the shaded region of the figures. The area of this region, however, is crucially dependent on the choice $m_{\tilde{e}_L} \approx m_{\tilde{e}_R}$. If $m_{\tilde{e}_R}$ is reduced, the shaded areas may shrink further. From Figs.[1A] and [1B] it is also apparent that the allowed region is almost independent of the choice of μ .

As discussed in the introduction, the processes **(A)** $e^+e^- \longrightarrow \tilde{\nu}\tilde{\nu}\gamma$, **(B)** $e^+e^- \longrightarrow \tilde{N}_1\tilde{N}_1\gamma$ and **(C)** $e^+e^- \longrightarrow \tilde{N}_1\tilde{N}_2\gamma$ contribute to the signal at $\sqrt{s} = 190$ GeV. At this energy the contribution of $e^+e^- \longrightarrow \tilde{N}_2\tilde{N}_2\gamma$ is indeed negligible. We have scanned over the entire LEP-1 allowed parameter space compatible with the VLSP scenario and have computed the cross sections from the processes **A** and **B**. If, for a particular choice of the parameters, \tilde{N}_2 is also a VLSP (the shaded region in Figs.[1A] and [1B]) then the contribution from the process **C** is also taken into account.

We show in Fig.[2A] the energy distribution of the photon for processes **A**(the solid line) and **B**(the dashed line). Photons in the forward direction (those emitted within an angle of 5° with the beam axis) are not considered. There is a mild cut $E_\gamma > 5$ GeV which in conjunction with the strong angular cuts (discussed below) removes other backgrounds including the ones from radiative Bhabha scattering with the final state e^+e^- pair going down the beam pipe. This cut also takes into account the detector threshold. The SUSY parameters used are $m_{\tilde{\nu}} = 50$ GeV, $m_{\tilde{e}_L} = m_{\tilde{e}_R}$, $M_{\tilde{g}} = 200$ GeV, $\mu = -200$ GeV and $\tan\beta = 5$.

The main SM background comes from the process **(D)** $e^+e^- \longrightarrow \nu\bar{\nu}\gamma$. The corresponding distribution (the dotted line) for this background is also shown in Fig.[2A]. As has already been discussed in [6], this distribution has a peak at about $(s - M_Z^2)/2\sqrt{s}$ corresponding to the decay of a real Z into $\nu\bar{\nu}$, as expected. Thus an upper cut of $E_\gamma < 60$ GeV optimises $\sigma = \frac{S}{\sqrt{B}}$ where S is the number of signal events and B is the number of background events.

In Fig.[2B] we present the angular distribution of the signal (for the above SUSY parameters) and the background following the conventions of Fig.[2A]. The distributions have similar characteristics. Thus angular cuts cannot further improve the quality of the signal. An irreducible background therefore remains.

In order to make a conservative assessment of the prospect of discovering the signal we impose an angular cut $40^\circ < \theta_\gamma < 140^\circ$, where θ_γ is the angle between the photon and the direction of the positron. This cut corresponds to the high p_T photons collected in the central part of the detector where photon detection efficiency is expected to be large (≈ 1). We have also studied the effects of a cut allowing for more angular coverage along with an explicit strong cut on the p_T of the photon to remove the Bhabha background. These cuts, introduced in a recent paper [15], are given by $18^\circ < \theta_\gamma < 162^\circ$, $1 < E_\gamma < 47.5$ GeV and $p_{T_\gamma} > 6.2$ GeV. Now the photons detected in the endcap region of the detector also contribute

and it is assumed that their detection efficiency is still large (≈ 1). We compare the response of the signal for the two sets of cuts in Table-I and find that they give very similar results. Using our cuts the background is 0.45 pb while for the cuts of [15] it is 0.51 pb.

In Fig.[3] we present the cross section as a function of $m_{\tilde{\nu}}$ for $m_{\tilde{e}_L} = m_{\tilde{e}_R}$ with our conservative cuts. The other SUSY parameters are varied within the following range: $200 \leq M_{\tilde{g}} \leq 400$, $-500 \leq \mu \leq 500$, $2 \leq \tan\beta \leq 30$. Only points allowed by LEP-1 data are considered. This scanning of the parameter space is more comprehensive than the one carried out in [6]. We note that $M_{\tilde{g}} < 212$ GeV is ruled out from SUSY searches at Tevatron for $m_{\tilde{q}} \approx M_{\tilde{g}}$ [4]. For $m_{\tilde{q}} \gg M_{\tilde{g}}$, the limit is $M_{\tilde{g}} > 144$ GeV. Strictly speaking these limits pertain to the pole mass of the gluino. However for $m_{\tilde{q}} \approx M_{\tilde{g}}$, the running gluino mass (the parameter more directly related to the chargino - neutralino sector through the assumption of unification) is equal to the pole mass to a very good approximation [22]. For $m_{\tilde{q}} \gg M_{\tilde{g}}$, the difference between the above two masses is significant. In fact it turns out that in this case the above limit on $M_{\tilde{g}}$ translates into a much relaxed bound on the running gluino mass.

It has also been qualitatively argued that these limits may be relaxed in the VLSP scenario[2]. However, no quantitative result exists. Moreover for low gluino masses the lighter chargino masses are also reduced and it becomes increasingly difficult to accommodate the VLSP scenario. We have, therefore, not assumed any drastic reduction of these mass limits and have taken conservatively $M_{\tilde{g}} \geq 200$ GeV. As discussed above our $M_{\tilde{g}}$ roughly corresponds to the running mass and is defined precisely at the beginning of this section. However the squark mass is essentially a free parameter in this model independent analysis and does not affect directly any of our numerical results. Sometimes therefore we shall take mainly for the purpose of illustration $M_{\tilde{g}}$ around 150 GeV which is allowed for heavy squarks. For $M_{\tilde{g}} \geq 400$ GeV the signal falls below the 2σ level and becomes uninteresting. The band within the solid lines in Fig.[3] corresponds to the combined cross sections σ_{tot} from the processes **A**, **B** and **D**, i.e. the scenario in which the $\tilde{\nu}$ is the only VLSP. In order to obtain conservative estimates, we have not considered the possibility that \widetilde{N}_2 may also be a VLSP. This is because the latter possibility can be evaded by an appropriate choice of $m_{\tilde{e}_R}$. The width of this band is due to varying $M_{\tilde{g}}$, μ and $\tan\beta$ within the above ranges and is a measure of SUSY parameter space consistent with the VLSP sceanrio for a given $m_{\tilde{\nu}}$. Taking into account the points where \widetilde{N}_2 is also a VLSP, for the choice $m_{\tilde{e}_L} = m_{\tilde{e}_R}$, the signal improves modestly due to the contribution from process **C** which is shown by

the band enclosed by the dashed lines. For comparison we also display the cross section for the background (process **D**) which corresponds to the lowest dotted horizontal line in the figure. The other two dotted horizontal lines correspond to 3σ and 5σ fluctuations of the background events for an integrated luminosity of 500 pb^{-1} . From the figure it may be noted that a 5σ signal can be obtained for $m_{\tilde{\nu}} \leq 52 \text{ GeV}$ for a very small region of SUSY parameter space. A much larger region of parameter space gives events above 3σ fluctuation with $m_{\tilde{\nu}} \leq 66 \text{ GeV}$. In this figure the upper edges of the bands correspond to $M_{\tilde{g}} = 200 \text{ GeV}$ and low $\tan\beta$ ($2 \leq \tan\beta \leq 6$). The cross section is rather insensitive to the variation of μ . For $M_{\tilde{g}} = 300 \text{ GeV}$ only 3σ signals can be obtained for a limited region of the parameter space while for $M_{\tilde{g}} = 400 \text{ GeV}$ the signal remains below the 2σ level for the entire region of the parameter space.

A clearer representation of the regions in the $(M_{\tilde{g}} - m_{\tilde{\nu}})$ plane that can be probed at $\sqrt{s}=190 \text{ GeV}$ is given by the contour plot in Fig.[4] for three values of $\tan\beta=2,10,30$. The points within the solid, dashed and dotted contours yield signals with statistical significances $\geq 4\sigma$, 3σ and 2σ respectively for suitable choices of μ . In this figure we have also considered signals in the region $150 \text{ GeV} \leq M_{\tilde{g}} \leq 200 \text{ GeV}$, since this region is still allowed by the Tevatron data for heavy squarks. The signal, however, is not considered for points where the VLSP constraints discussed above are not satisfied.

Since experiments at LEP-1.5 are in progress, the cross section at $\sqrt{s}=130 \text{ GeV}$ is of considerable interest. However, even for $m_{\tilde{\nu}}=50 \text{ GeV}$ and other favourable choices of the SUSY parameters ($\mu = -300$, $M_{\tilde{g}}=200$ and $\tan\beta=5$) the cross section happens to be rather disappointing. The total cross section of the processes **A–C** is 0.054 pb while the background is 0.373 pb with the cuts $5 < E_{\gamma} < 20 \text{ GeV}$ and $40^\circ < \theta_{\gamma} < 140^\circ$. Thus for an integrated luminosity of 6 pb^{-1} the statistical significance is certainly $< 3\sigma$.

From the above results it is clear that the process under consideration has a rather modest cross section at LEP. The bulk of the constraints in the chargino-sneutrino sector in the VLSP scenario is therefore likely to come from direct chargino searches [7]. Nevertheless this process is likely to play a complementary role in the regions of the parameter space where the chargino and the sneutrino are nearly degenerate and the sneutrino masses are relatively small. To illustrate this point we consider an example with $\mu = -350 \text{ GeV}$, $\tan\beta = 6$ and $M_{\tilde{g}} = 150 \text{ GeV}$. In this case the chargino mass is 53 GeV . The total signal cross section for $m_{\tilde{\nu}} = 50 \text{ GeV}$ is 0.17 pb which corresponds to 5σ . It can be readily checked

that the signal is weaker for higher $M_{\tilde{g}}$ s, i.e., for larger chargino - sneutrino mass differences. This reduction for higher $M_{\tilde{g}}$ is essentially due to propagator suppression in process A) and due to kinematical effects in processes B) and C) and holds for other choices of SUSY parameters. Thus in contrast to the direct chargino searches, the regions of the parameter space where the chargino and the sneutrino are nearly mass degenerate can be probed via this mode, provided the sneutrinos are not too heavy. In Fig.4 the shaded region corresponds to $m_{\tilde{\nu}} < M_{\tilde{\chi}_1^\pm} < m_{\tilde{\nu}} + 5 \text{ GeV}$, where the chargino decay is likely to be difficult to detect. The statistical significance of the signal cannot be judged directly from this figure for the entire shaded region. This is because we have not computed the cross sections for $M_{\tilde{g}} < 150 \text{ GeV}$, since this region is either ruled out or marginally allowed by the Tevatron data depending on the assumption on the squark mass. The cross sections have also not been computed for parameters for which the VLSP condition is not satisfied. However in regions not excluded by these considerations, the cross section is significant. This is especially so for relatively large $\tan\beta$.

3 The Signal at NLC

In this section we discuss the signal and the background for e^+e^- collisions at **NLC** for two values of centre of mass energy *viz.* $\sqrt{s}=350 \text{ GeV}$ and $\sqrt{s}=500 \text{ GeV}$.

For $\sqrt{s}=350 \text{ GeV}$ the energy and angular distributions of the radiated photon in the signal [process **(A)** and process **(B)**] and background [process **(D)**] are shown in Fig.[5A] and Fig.[5B] respectively. The set of SUSY parameters used are $m_{\tilde{\nu}}=80 \text{ GeV}$ and $M_{\tilde{g}}=350$, $\mu = -500$ and $\tan\beta=5$. The conventions for different curves are the same as those in Figs.[2A] and [2B]. The energy distribution (Fig.[5A]) of the background has a peak at about $\sqrt{s}/2$ which is the beam energy. Thus an upper cut of $E_\gamma < 150 \text{ GeV}$ is set.

A lower cut on the photon energy is set from naive kinematics of the background due to radiative Bhabha scattering which requires special care [18]. We have devised our cuts against this background assuming that e^+e^- scattered within a cone of 10° with respect to the beam axis may remain undetected. From kinematics we find that in this situation the photons are restricted by the following criteria: $E_\gamma \leq 65 \text{ GeV}$, $p_{T_\gamma} \leq 52 \text{ GeV}$. We have also checked that either of the above cuts reduces the Bhabha background completely. From the E_γ distribution (Fig.[5A]) it is also clear that a strong lower cut of $E_\gamma > 65 \text{ GeV}$ reduces

both the signal and the $\nu\bar{\nu}$ background from the process **(D)**, but does not affect the $\sigma = \frac{S}{\sqrt{B}}$ ratio drastically. The anticipated high luminosity ($\mathcal{L} \sim 10^{33} \text{cm}^{-2} \text{sec}^{-1} \sim 3 \times 10^4 \text{pb}^{-1}$ over a year) [17] ensures that a respectable number of events is obtained in spite of this reduction due to stiff cuts.

Another set of cuts, subjected to a rather strong assumption about the detectors, was discussed in [18]. In particular it was assumed that it is possible to detect in a radiative Bhabha event the scattered e^+ or e^- emitted at an angle $\theta_{min} < \theta_e < 10^\circ$ ($\theta_{min} \approx 1.6^\circ$ at $\sqrt{s}=350$ GeV). If this indeed is the case then the above stringent lower p_T cut on the photon can be significantly relaxed. We shall compare below the effects of these two sets of cuts. It turns out that with the milder cuts of [18] a larger region of the parameter space can be probed.

From Fig.[5B] it is clear that the angular distributions for the signal and background processes have similar characteristics. Thus, as in section 2, angular cuts cannot improve the quality of the signal. Nevertheless we impose conservatively angular cuts of $40^\circ < \theta_\gamma < 140^\circ$ which correspond to the central region of the detector where photon detection efficiency is expected to be very high (≈ 1).

In Fig.[6] we present the cross section as a function of $m_{\tilde{\nu}}$. The conventions are the same as those in Fig.[3] for the bands and the horizontal lines. The dashed band contains the additional contributions from $\widetilde{N}_1\widetilde{N}_2$ and $\widetilde{N}_2\widetilde{N}_2$ pairs at points where \widetilde{N}_2 is also a VLSP. As emphasised in section 2, the latter contributions will be absent if $m_{\tilde{e}_{L,R}} < \widetilde{N}_2$. As $m_{\tilde{\nu}}$ increases the minimum $M_{\tilde{g}}$ which can accommodate the VLSP scenario also increases (see Fig.[1]). For example, at $m_{\tilde{\nu}}=100$ GeV, only $M_{\tilde{g}} \geq 375$ GeV are consistent with the VLSP scenario. In addition to the obvious kinematical effects, suppressions due to $\tilde{\chi}^\pm$ and $\tilde{e}_{L,R}$ propagators, therefore, tend to decrease the signal with increasing $m_{\tilde{\nu}}$. Also, for larger $M_{\tilde{g}}$, the contributions from $\widetilde{N}_i\widetilde{N}_j$ pairs decrease due to kinematic effects. The reduction of the total VLSP cross section with increasing $m_{\tilde{\nu}}$ is therefore a complicated combination of several effects. It is clearly seen from Fig.[6] that the SUSY parameter space consistent with the VLSP scenario, indicated by the width of the bands, gradually shrinks as $m_{\tilde{\nu}}$ increases.

We find that for $m_{\tilde{\nu}} \leq 110$ GeV a 5σ signal can be obtained even with our conservative cuts without imposing any special requirement on the detectors. This unfortunately is much smaller than the kinematic limit at $\sqrt{s}=350$ GeV. It is therefore worthwhile to study the effects of the relaxed cuts proposed in [18]. We compare the efficiencies of the two sets of cuts

in Table-II. The complex interplay between the $m_{\tilde{\nu}}$ and $M_{\tilde{g}}$ in the VLSP scenario, discussed in the last paragraph, is also clearly exhibited in Table-II which is drawn for $M_{\tilde{g}}=400$ GeV. For this $M_{\tilde{g}}$, the \widetilde{N}_2 is not a VLSP for $m_{\tilde{\nu}}=110$ and 125 GeV, which leads to sleptons lighter than the \widetilde{N}_2 . The cross section is, therefore, larger for heavier sneutrinos at this $M_{\tilde{g}}$. It follows from this table that significantly larger regions of the parameter space can be scanned if improvement in instrumentation discussed in [18] allows the scattered e^+e^- in a radiative Bhabha event to be tracked down in the beam pipe.

In Figs.[7] we present contour plots in the $(M_{\tilde{g}} - m_{\tilde{\nu}})$ plane that can be probed at $\sqrt{s}=350$ GeV for three values of $\tan\beta$, $\tan\beta=2,10,30$. In these, the dotted(outermost) contours represent the areas in the $(M_{\tilde{g}} - m_{\tilde{\nu}})$ plane where a $\geq 3\sigma$ signal can be obtained. The dashed(middle ones) and the solid(innermost ones) show the same for 4σ and 5σ signals respectively. As $m_{\tilde{\nu}}$ increases, a distinct rise in the lowest allowed $M_{\tilde{g}}$ is also a very indicative feature of the VLSP scenario.

The photon energy and angular distributions for $\sqrt{s}=500$ GeV are shown in Fig.[8A] and Fig.[8B] respectively. The conventions and features of the curves are similar to ones for $\sqrt{s}=350$ GeV case. An upper cut of $E_\gamma < 225$ GeV is set. From kinematical considerations a strong lower cut of $E_\gamma > 95$ GeV is imposed to eliminate completely the radiative Bhabha background. Along with this an angular cut of $40^\circ < \theta_\gamma < 140^\circ$ corresponding to the central region of the detector is imposed.

In Fig.[9] we present the cross section as a function of $m_{\tilde{\nu}}$. The conventions are the same as in Fig.[6]. We find that only for $m_{\tilde{\nu}} \leq 125$ GeV 5σ signals can be obtained using our conservative cuts and optimistic choices of SUSY parameters. It is again much smaller than the kinematic limit at $\sqrt{s}=500$ GeV. Once again by using the relaxed cuts proposed in [18], the search limit can be significantly increased. We compare the efficiencies of the two cuts in Table-III which shows the prospect of improvement in the search limit if the relaxed cuts [18] are permissible due to improvements in detector designs.

In Figs.[10] we present the contour plots at $\sqrt{s}=500$ GeV for three values of $\tan\beta$. The conventions are exactly the same as in Fig.[7].

To end this section it is noteworthy that a greater region in the $(m_{\tilde{\nu}} - M_{\tilde{g}})$ plane can be probed at $\sqrt{s}=500$ GeV with appreciable statistical significance compared to the $\sqrt{s}=350$ GeV case, as expected. However, this gain is not commensurate with the increase in beam energy. Also the searches at NLC via this mode will be very effective in constraining the

regions of the parameter space which are in principle accessible to direct chargino searches at LEP 2 energies but can not be probed there due to near mass degeneracy of the chargino and the sneutrino and relatively large mass of the sneutrino.

4 The Signal in $N=1$ SUGRA Models

In this section we consider a more constrained scenario based on $N=1$ SUGRA with a common scalar mass (m_0) at the GUT scale [19]. It should, however, be noted that recently many viable models with non-universal scalar masses have been constructed [20]. Yet models with a common m_0 continue to be popular and its implication for the VLSP scenario is worth investigating. However, no assumption about the Higgs sector and, consequently, about the $SU(2) \otimes U(1)$ breaking mechanism is made. As pointed out in [7] the VLSP scenario can also be accommodated in this more restrictive model. It was shown that the VLSP constraints require a relatively light gluino with $M_{\tilde{g}}$ bounded by the relation

$$m_{1/2} \leq 1.4\sqrt{D_Z}$$

where $m_{1/2}$ is the common gaugino mass at the GUT scale and D_Z has been defined earlier. This bound also restricts the masses of $\tilde{\chi}_1^\pm$ and \tilde{N}_2 severely. Since in the VLSP scenario the sneutrino has to be lighter than the above particles, $m_{\tilde{\nu}}$ is also bounded from above. As a consequence, this scenario can be tested conclusively at relatively low energy machines, e.g., at an e^+e^- collider $\sqrt{s}=350$ GeV. Sizable cross sections may be obtained at $\sqrt{s}=190$ GeV provided $m_{\tilde{\nu}}$ happens to be in the lower part of its allowed range.

Here we consider the allowed region of the $(m_0 - m_{1/2})$ mass plane in the VLSP scenario as given in [7]. Using the formulae in [7] one can calculate the sparticle masses and hence the cross sections at various points of the above region using the cuts stated in the earlier sections. We present some of the sample results at $\sqrt{s}=190$ GeV and $\sqrt{s}=350$ GeV in Table-IV. It is seen from Table-IV that the entire region of the parameter space allowed in the VLSP scenario gives an observable signal ($\geq 5\sigma$) at $\sqrt{s}=350$ GeV. In each case, it turns out that $m_{\tilde{q}}$ is nearly equal to $M_{\tilde{g}}$. Using the bounds of [4] we have restricted ourselves to the cases with $m_{\tilde{q}} = M_{\tilde{g}} \approx 200$ GeV.

If one further assumes radiative breaking of $SU(2) \otimes U(1)$ symmetry then the number of free parameters reduces further. In particular μ becomes a fixed parameter, apart from a sign

ambiguity, for given m_0 , $m_{1/2}$, $\tan\beta$ and m_t . We have already seen that the cross sections are not very sensitive to μ . We therefore work with the representative choice $\mu = -M_{\tilde{g}}$ used by other authors [21]. The allowed regions of [7] now reduce to narrow strips. In Table[IV] we give the cross sections at a few representative points and note that observable signals with high statistical significances are predicted at $\sqrt{s}=350$ GeV.

5 Radiative Corrections

In this section we briefly consider the radiative corrections to the cross sections. We follow the structure function approach of [16]. We, however, restrict ourselves to the corrections due to soft photon brehmstrahlung only to all orders in perturbation theory. The formula for the corrected cross section can be found in equation(18) of the fourth paper of [16]. In this formula we have substituted the cross sections given in the Appendix. At LEP-2 energies the background reduces from 0.45 pb to 0.34 pb. The changes in the signal cross sections are shown in Table-V. For each sneutrino mass the cross sections with and without the radiative correction are presented using CutA of Table-I. It is seen that both the signal and the background reduce sizably due to this correction but the ratio σ remains almost unaffected. It may also be noted that inspite of this reduction the total number of signal events remains adequate for a luminosity of 500 pb^{-1} .

6 Conclusions

Following our earlier works [2, 5, 6, 7], we have emphasised in this paper that currently popular search strategies for supersymmetric particles may be significantly affected in the VLSP scenario with relatively light sneutrinos and the second lightest neutralino, which may decay dominantly into invisible channels, leading to two extra carriers of missing energy (in addition to the lightest supersymmetric particle (LSP)).

We have focussed our attention on the processes (a) $e^+e^- \longrightarrow \tilde{\nu}\tilde{\nu}\gamma$, (b) $e^+e^- \longrightarrow \tilde{N}_i\tilde{N}_j\gamma$ ($i,j=1,2$) and (c) $e^+e^- \longrightarrow \nu\bar{\nu}\gamma$, all of which contribute to $e^+e^- \longrightarrow \gamma + \cancel{E}$ in the VLSP scenario. Formulae for the cross sections taking into account full mixings of the charginos and the neutralinos are derived and presented in the Appendix.

It is shown that for suitable choices of SUSY parameters at LEP-2 energies, process (a)

contributes dominantly to the signal. The contribution from $\widetilde{N}_1 \widetilde{N}_2$ pairs is also comparable to that from LSP pairs, which alone produces the signal in the conventional MSSM. This leads to the interesting possibility that at LEP-2 the VLSP scenario can be distinguished not only from the SM but also from the conventional MSSM. If the sneutrino masses happen to be in a small but experimentally interesting range $m_{\tilde{\nu}} = 45 - 55$ GeV and the gluinos are relatively light ($M_{\tilde{g}} \approx 200$ GeV), sneutrino and neutralino pairs (processes (a) and (b)) may contribute significantly to the signal as an enhancement over the SM (process (c)) or the conventional MSSM (process (b) with $i = j = 1$) predictions. After suitable kinematical cuts [6, 15], the statistical significance of the signal depends crucially on the choice of SUSY parameters and may be $\geq 5\sigma$ for $m_{\tilde{\nu}}$ and $M_{\tilde{g}}$ as above.

It is further shown that a much larger region of the parameter space can be scanned at a high luminosity e^+e^- collider at 350 and 500 GeV like the proposed NLC machine even with conservative cuts. If further improvements in detector design [18] allow a relaxation of these strong cuts, even larger regions of the parameter space can be scanned. At $\sqrt{s}=350(500)$ GeV, $m_{\tilde{\nu}} \approx 150(180)$ GeV may be probed for $M_{\tilde{g}}$ in the range 400–550 GeV using the cuts of [18]. This signal is viable even if the mass splitting between the chargino and the sneutrino is small and thus, can play a complementary role to direct chargino searches at LEP-2 and NLC. The latter process can probe larger regions of the parameter space, but the signal may disappear if the above mass splitting is small. The signal remains observable even in the context of more restricted models based on $N=1$ SUGRA with common scalar and gaugino masses at a high scale. The effect of soft photon emissions to all orders in perturbation theory is also discussed briefly using the approach of [16].

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Appendix

In this appendix we systematically present the relevant formulae for calculating the cross sections of different processes. Throughout this paper we use the following Standard Model Parameters :

$\alpha = 1/128.8$, $G_F = 1.16637 \times 10^{-5}$, $M_Z = 91.187 \text{ GeV}$, $M_W = 80.22 \text{ GeV}$, $\Gamma_Z = 2.498 \text{ GeV}$, $\Gamma_W = 2.25 \text{ GeV}$, $T_3^e = -0.5$, $Q_e = -1$, $S_W^2 = \sin^2 \theta_W = 0.232$, $C_W = \cos \theta_W$, $C_V = 2T_3^e - 4Q_e S_W^2$, $C_A = -2T_3^e$, $C_L = C_V - C_A$, $S_{avg} = 1/4$ (spin averaging over the initial spin configuration), $F_{ovl} = 128\pi\alpha G_F^2 M_W^4$.

A : The process $e^+ e^- \longrightarrow \nu \bar{\nu} + \gamma$.

We label the particles by the following indices: $e^+ \Rightarrow 1$, $e^- \Rightarrow 2$, $\nu \Rightarrow 3$, $\bar{\nu} \Rightarrow 4$, $\gamma \Rightarrow 5$.

We have used the following abbreviations : $P_{ij} = p_i \cdot p_j$, $B_W = \frac{1}{(2P_{34} - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$, $B = 2P_{34} - M_Z^2$, $W_3 = -(2P_{14} + M_W^2)$, $W_4 = -(2P_{23} + M_W^2)$, $W_{B3} = W_3^2 + (M_W \Gamma_W)^2$, $W_{B4} = W_4^2 + (M_W \Gamma_W)^2$, $\epsilon(ijkl) = \epsilon_{\alpha\beta\gamma\delta} P_i^\alpha P_j^\beta P_k^\gamma P_l^\delta$, where p_i is the momentum of the i -th particle.

In the following $\mathbf{T}_{ij} = A_i A_j^\dagger + \text{H.C.}$, where A_i is the amplitude of the i -th Feynman Diagram apart from an overall factor F_{ovl} defined at the beginning of this appendix. In this sub-section we consider only the diagrams (Fig.[11]) contributing to the cross section of the process $e^+ e^- \longrightarrow \nu \bar{\nu} + \gamma$.

The relevant matrix element squared can be computed from the following formulae:

$$\mathbf{T}_{11} = \frac{B_W}{4C_W^4 P_{25}} \left[(C_V^2 + C_A^2) U_{11} + 2C_V C_A V_{11} \right],$$

$$\mathbf{U}_{11} = P_{35}(P_{12} - 2P_{13} - P_{15}) + P_{13}(P_{15} + P_{25}),$$

$$\mathbf{V}_{11} = P_{35}(P_{12} - P_{15}) - P_{13}(P_{15} + P_{25}).$$

$$\mathbf{T}_{12} = \frac{-B_W}{4C_W^4 P_{15} P_{25}} \left[(C_V^2 + C_A^2) U_{12} + 2C_V C_A V_{12} \right],$$

$$\begin{aligned} \mathbf{U}_{12} = & -2P_{12}^2(P_{13} + P_{23} - P_{35}) + 2P_{12}P_{13}(P_{15} + 2P_{23} + P_{25} - P_{35}) - P_{12}(P_{15} + P_{25})(P_{35} - 2P_{23}) \\ & + (P_{13}P_{25} - P_{15}P_{23})(2P_{13} + P_{15} - 2P_{23} - P_{25}) - 2P_{12}P_{23}P_{35}, \end{aligned}$$

$$\begin{aligned} \mathbf{V}_{12} = & 2P_{12}^2(P_{13} - P_{23}) - 2P_{12}P_{13}(P_{15} + 2P_{25}) + P_{12}P_{15}(4P_{23} - P_{35}) + P_{12}P_{25}(2P_{23} + P_{35}) \\ & + (P_{15} + P_{25})(P_{13}P_{25} - P_{15}P_{23}). \end{aligned}$$

$$\mathbf{T}_{13} = -\frac{2B_W C_L}{C_W^2 P_{25} W_{B3}} (BW_3 + M_W M_Z \Gamma_W \Gamma_Z) \left[P_{13}(P_{15} + P_{25} - P_{35}) \right].$$

$$\mathbf{T}_{14} = \frac{-B_W C_L}{C_W^2 P_{15} P_{25} W_{B4}} \left[(BW_4 + M_W M_Z \Gamma_W \Gamma_Z) R_{14} + (M_Z \Gamma_Z W_4 - M_W \Gamma_W B) I_{14} \right],$$

$$\mathbf{R}_{14} = P_{13}(P_{14}P_{25} - P_{12}P_{45}) - P_{13}P_{24}(P_{15} + P_{25}) - P_{12}P_{24}(P_{35} - 2P_{13}) + P_{15}P_{23}P_{24},$$

$$\mathbf{I}_{14} = -\epsilon(3125)P_{24} + \epsilon(4125)P_{13}.$$

$$\begin{aligned} \mathbf{T}_{15} = & \frac{B_W C_L}{C_W^2 P_{25} W_{B3} W_{B4}} \left[\left\{ B(W_3 W_4 - M_W^2 \Gamma_W^2) + M_Z M_W \Gamma_Z \Gamma_W (W_3 + W_4) \right\} R_{15} \right. \\ & \left. - \left\{ B(W_3 + W_4) M_W \Gamma_W - M_Z \Gamma_Z (W_3 W_4 - M_W^2 \Gamma_W^2) \right\} I_{15} \right], \end{aligned}$$

$$\begin{aligned} \mathbf{R}_{15} = & (3P_{24} - P_{45})(P_{15}P_{23} - P_{12}P_{35}) - P_{13}P_{24}(P_{15} - 2P_{23} + 2P_{24} + 2P_{25} - 2P_{45} - 2P_{12}) \\ & + P_{14}P_{25}(P_{13} - P_{23}) - P_{13}P_{45}(P_{12} - 3P_{25} + 2P_{23}) + P_{12}P_{25}P_{34}, \end{aligned}$$

$$\mathbf{I}_{15} = -\epsilon(3425) \left[P_{12} + P_{15} \right] + \epsilon(4125) \left[P_{13} - P_{23} - P_{35} \right] - 2\epsilon(3125)(P_{24} - P_{45}) + \epsilon(3415)P_{25}$$

$$\mathbf{T}_{22} = \frac{B_W}{4C_W^4 P_{15}} \left[(C_V^2 + C_A^2) U_{22} + 2C_V C_A V_{22} \right],$$

$$\mathbf{U}_{22} = (P_{12} - 2P_{23} - P_{25})P_{35} + (P_{15} + P_{25})P_{23},$$

$$\mathbf{V}_{22} = (P_{25} - P_{12})P_{35} + (P_{15} + P_{25})P_{23}.$$

$$\mathbf{T}_{23} = \frac{-B_W C_L}{C_W^2 P_{15} P_{25} W_{B3}} \left[(BW_3 + M_W M_Z \Gamma_W \Gamma_Z) R_{23} + (M_Z \Gamma_Z W_3 - M_W \Gamma_W B) I_{23} \right],$$

$$\mathbf{R}_{23} = P_{13}P_{25}(P_{14} - P_{24}) - P_{15}P_{24}(P_{13} - P_{23}) + P_{12}P_{24}(2P_{13} - P_{35}) - P_{12}P_{13}P_{45},$$

$$\mathbf{I}_{23} = \epsilon(3125)P_{24} - \epsilon(4125)P_{13}.$$

$$\mathbf{T}_{24} = \frac{-2B_W C_L}{C_W^2 P_{15} W_{B4}} (B W_4 + M_W M_Z \Gamma_W \Gamma_Z) [P_{35}(P_{12} - P_{23} - P_{25})].$$

$$\begin{aligned} \mathbf{T}_{25} = \frac{-B_W C_L}{C_W^2 P_{15} W_{B3} W_{B4}} & \left[\left\{ B(W_3 W_4 - M_W^2 \Gamma_W^2) + M_Z M_W \Gamma_Z \Gamma_W (W_3 + W_4) \right\} R_{25} \right. \\ & \left. - \left\{ B(W_3 + W_4) M_W \Gamma_W - M_Z \Gamma_Z (W_3 W_4 - M_W^2 \Gamma_W^2) \right\} I_{25} \right], \end{aligned}$$

$$\mathbf{R}_{25} = -P_{12} P_{13} (P_{34} - P_{45}) - P_{12} P_{35} (P_{24} - P_{34} - P_{45}) + P_{13} P_{14} (P_{23} - 2P_{24} - P_{25})$$

$$+ P_{13} P_{24} (P_{13} + P_{15} + P_{25} - P_{35} + P_{45}) - P_{14} P_{35} (P_{23} - P_{24} + P_{25}) - P_{15} P_{24} (P_{23} + P_{34} + 3P_{35}),$$

$$\mathbf{I}_{25} = \epsilon(3415)(P_{13} + P_{24}) + \epsilon(4125)(P_{13} + P_{35}) - \epsilon(3125)P_{24} - \epsilon(3412)P_{35}.$$

$$\mathbf{T}_{33} = \frac{4}{P_{25} W_{B3}} [P_{13}(P_{15} + P_{25} - P_{35})].$$

$$\mathbf{T}_{34} = \frac{4}{P_{15} P_{25} W_{B3} W_{B4}} [(W_3 W_4 + M_W^2 \Gamma_W^2) R_{34} - M_W \Gamma_W (W_3 - W_4) I_{34}],$$

$$\begin{aligned} \mathbf{R}_{34} = P_{12} P_{13} (2P_{12} - 2P_{15} - 2P_{23} - 3P_{25} + P_{35}) - P_{12} P_{35} (P_{12} - P_{23} - P_{25}) \\ - (P_{13} - P_{23} - P_{25})(P_{13} P_{25} - P_{15} P_{23}) + P_{12} P_{15} P_{23}, \end{aligned}$$

$$\mathbf{I}_{34} = -\epsilon(3125)(P_{12} + P_{13} - P_{23} - P_{25}).$$

$$\mathbf{T}_{35} = \frac{-4}{P_{25} W_{B3} W_{B4}} [W_4 R_{35} - M_W \Gamma_W I_{35}],$$

$$\begin{aligned} \mathbf{R}_{35} = -P_{12} P_{35} (3P_{12} + P_{13} - P_{15} - 3P_{23} - 3P_{25} + P_{35}) + P_{12} P_{13} (4P_{23} + P_{25}) + \\ P_{15} P_{23} (3P_{12} - 3P_{13} - P_{15} - 3P_{23} \\ - 3P_{25} + P_{35}) + P_{13} P_{25} (P_{13} + 3P_{15} - 5P_{23} + 3P_{25} - 3P_{35}) - 4P_{13} P_{23} (P_{23} - P_{35}), \end{aligned}$$

$$\mathbf{I}_{35} = -(3P_{12} + P_{13} - P_{15} - 3P_{23} - 3P_{25} + P_{35}) \cdot \epsilon(3125)$$

$$\mathbf{T}_{44} = \frac{-4}{P_{15} W_{B4}} [P_{35}(P_{12} - P_{23} - P_{25})].$$

$$\mathbf{T}_{45} = \frac{8}{P_{15}W_{B3}W_{B4}}[W_3R_{45} - M_W\Gamma_W I_{45}],$$

$$\mathbf{R}_{45} = -P_{12}P_{13}(P_{12} - P_{13} - 2P_{15} - P_{23} - 2P_{25} + P_{35}) - P_{12}P_{15}(P_{23} + P_{35}) - \\ P_{13}P_{23}(2P_{13} + 2P_{15} + P_{25} - 2P_{35})$$

$$-P_{13}P_{25}(P_{13} + P_{15} + P_{25} - 2P_{35}) + P_{15}P_{23}(P_{23} + P_{25} + 2P_{35}) - P_{35}(P_{12}P_{35} - 2P_{15}P_{25}),$$

$$\mathbf{I}_{45} = -(P_{12} + P_{13} - P_{23} - P_{25})\epsilon(3125).$$

$$\mathbf{T}_{55} = \frac{-4}{W_{B3}W_{B4}}[U_{55}],$$

$$\mathbf{U}_{55} = -P_{12}P_{23}(P_{12} + 6P_{13} + 5P_{15} - P_{23} - P_{25} + 4P_{35}) + P_{12}P_{35}(3P_{12} + P_{13} - 3P_{15} - 3P_{25} - P_{35})$$

$$-P_{13}P_{25}(P_{12} + 3P_{13} + 3P_{15} - 8P_{23} + 3P_{25} - 7P_{35}) - P_{13}P_{23}(P_{13} - 5P_{15} - 7P_{23} + 3P_{35}) \\ + 2P_{15}P_{23}(P_{15} + 3P_{23} + 3P_{25} + P_{35}) + 4P_{15}P_{25}P_{35}.$$

$$\begin{aligned} T(\nu\bar{\nu}) &= 3(T_{11} + T_{12} + T_{22}) + T_{13} + T_{14} + T_{15} \\ &+ T_{23} + T_{24} + T_{25} + T_{33} + T_{34} + T_{35} \\ &+ T_{44} + T_{45} + T_{55} \end{aligned}$$

The differential cross section is given by

$$d\sigma = F_{ovl}S_{avg} \frac{T(\nu\bar{\nu})}{64E_{CM}^2\pi^5} \delta^4(p_1 + p_2 - \sum_{i=3}^5 p_i) \prod_{i=3}^5 \frac{d^3p_i}{2E_i}$$

Other cross sections are obtained by replacing the T -factor in the above formula by the appropriate expressions calculated in the following appendices.

B : The process $e^+ e^- \longrightarrow \tilde{\nu}\tilde{\nu} + \gamma$.

In this subsection we consider the diagrams (Fig.[12]) contributing to the cross section of the process $e^+ e^- \longrightarrow \tilde{\nu}\tilde{\nu} + \gamma$. We define for this subsection:

$$B_W = \frac{1}{\left\{2(m_{\tilde{\nu}}^2 + P_{34}) - M_Z^2\right\}^2 + (M_Z G_Z)^2}.$$

We label the particles by the following indices: $\tilde{\nu} \Rightarrow 3, \tilde{\bar{\nu}} \Rightarrow 4$, while the indices 1,2 and 5 have the same meaning as in the previous subsection.

The convention for the T_{ij} -s in this subsection is the same as in the last subsection for the process under consideration.

$$\mathbf{T}_{11} = \frac{3B_W}{8C_W^4 P_{25}} (C_V^2 + C_A^2) \left[(P_{13} - P_{14})(P_{35} - P_{45}) + P_{15}(P_{34} - m_{\tilde{\nu}}^2) \right].$$

$$\mathbf{T}_{12} = \frac{-3B_W}{8C_W^4 P_{15} P_{25}} (C_V^2 + C_A^2) U_{12},$$

$$\mathbf{U}_{12} = P_{12} \left\{ 2(P_{12} - P_{15} - P_{25})(m_{\tilde{\nu}}^2 - P_{34}) - (P_{13} - P_{14})(2P_{23} - 2P_{24} - P_{35} + P_{45}) + (P_{23} - P_{24})(P_{35} - P_{45}) \right\}$$

$$+ P_{15} \left\{ P_{23}(P_{13} - P_{14} - P_{23} + 2P_{24}) - P_{24}(P_{13} - P_{14} + P_{24}) \right\} - P_{25} \left\{ P_{13}(P_{13} - 2P_{14} - P_{23} + P_{24}) + P_{14}(P_{14} + P_{23} - P_{24}) \right\}.$$

$$\mathbf{T}_{13} = -\frac{B_W C_L}{2C_W^2 P_{25}} (M_Z^2 - 2m_{\tilde{\nu}}^2 - 2P_{34}) \sum_{a=1}^2 \frac{|\mathcal{V}_{a1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})} [U_{13}],$$

$$\mathbf{U}_{13} = -P_{12}(2P_{25} - 3P_{35}) + P_{13}(3P_{25} - 4P_{35}) + P_{15}(2m_{\tilde{\nu}}^2 - 3P_{23})$$

where \mathcal{V}_{a1} , $a=1,2$ are the mixing factors corresponding to the two charginos.

$$\mathbf{T}_{14} = -\frac{B_W C_L}{2C_W^2 P_{15} P_{25}} (M_Z^2 - 2m_{\tilde{\nu}}^2 - 2P_{34}) \sum_{a=1}^2 \frac{|\mathcal{V}_{a1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})} [U_{14}],$$

$$\mathbf{U}_{14} = 2(m_{\tilde{\nu}}^2 P_{12} - P_{13} P_{23})(P_{12} - P_{15} - P_{25}) + P_{12} P_{35}(2P_{13} + 2P_{23} - P_{25}) - P_{13} P_{25}(2P_{13} - P_{25}) - P_{15} P_{23}(2P_{23} + P_{25}) - 2P_{12} P_{13} P_{23}.$$

$$\mathbf{T}_{15} = -\frac{B_W C_L}{C_W^2 P_{25}} (M_Z^2 - 2m_{\tilde{\nu}}^2 - 2P_{34}) \sum_{a=1}^2 \frac{|\mathcal{V}_{a1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})} [2U_{15} - m_a^2 W_{15}],$$

$$\mathbf{U}_{15} = P_{23} \left\{ m_{\tilde{\nu}}^2 (P_{15} - P_{12}) - P_{25} (P_{12} - 3P_{13} - P_{15}) + 2P_{13} (P_{23} - P_{35}) \right\},$$

$$\mathbf{W}_{15} = P_{12}(P_{25} - P_{35}) - P_{13} P_{25} + P_{15} P_{23}.$$

$$\mathbf{T}_{22} = \frac{3B_W}{4C_W^4 P_{15}} (C_V^2 + C_A^2) \left[(P_{23} - P_{24})(P_{35} - P_{45}) + P_{25}(P_{34} - m_{\tilde{\nu}}^2) \right].$$

$$\begin{aligned}\mathbf{T}_{23} &= -\frac{-B_W C_L}{2C_W^2 P_{15} P_{25}} (M_Z^2 - 2m_{\tilde{\nu}}^2 - 2P_{34}) \sum_{a=1}^2 \frac{|\mathcal{V}_{a1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})} [U_{23}] , \\ \mathbf{U}_{23} &= -P_{12} \left\{ 2m_{\tilde{\nu}}^2 (P_{12} - P_{15} - P_{25}) - 2P_{13} (2P_{23} - P_{35}) + P_{15} (2P_{25} - P_{35}) + 2P_{35} (P_{23} - P_{25}) \right\} \\ &\quad + (P_{13} P_{25} - P_{15} P_{23}) (2P_{13} + P_{15} - 2P_{23} - 2P_{25}).\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{24} &= \frac{B_W C_L}{2C_W^2 P_{15}} (M_Z^2 - 2m_{\tilde{\nu}}^2 - 2P_{34}) \sum_{a=1}^2 \frac{|\mathcal{V}_{a1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})} [U_{24}] , \\ \mathbf{U}_{24} &= P_{25} (P_{13} - 2m_{\tilde{\nu}}^2) - P_{35} (P_{12} - 4P_{23}) - P_{15} P_{23}.\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{25} &= \frac{B_W C_L}{C_W^2 P_{15}} (M_Z^2 - 2m_{\tilde{\nu}}^2 - 2P_{34}) \sum_{a=1}^2 \frac{|\mathcal{V}_{a1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})} [2U_{25} - m_a^2 W_{25}] , \\ \mathbf{U}_{25} &= m_{\tilde{\nu}}^2 \left\{ P_{12} (P_{12} - P_{13} - P_{15} - P_{25}) + P_{25} (P_{13} + P_{15}) \right\} - P_{23} (2P_{13} + P_{15} - 2P_{35}) \\ &\quad (P_{12} - P_{13} - P_{15}),\end{aligned}$$

$$\mathbf{W}_{25} = P_{12} (P_{15} - P_{35}) + P_{13} P_{25} - P_{15} P_{23}.$$

$$\mathbf{T}_{33} = \frac{4}{P_{25}} \sum_{a=1}^2 \sum_{b=1}^2 \frac{|\mathcal{V}_{a1}|^2 |\mathcal{V}_{b1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{14})} [2P_{14} P_{45} - m_{\tilde{\nu}}^2 P_{15}] .$$

$$\begin{aligned}\mathbf{T}_{34} &= \frac{4}{P_{15} P_{25}} \sum_{a=1}^2 \sum_{b=1}^2 \frac{|\mathcal{V}_{a1}|^2 |\mathcal{V}_{b1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{23})} [U_{34}] , \\ \mathbf{U}_{34} &= P_{12} \left\{ -m_{\tilde{\nu}}^2 (P_{12} - P_{15} - P_{25}) + P_{13} (2P_{23} - P_{35}) - P_{35} (P_{23} - P_{25}) \right\} \\ &\quad + (P_{13} P_{25} - P_{15} P_{23}) (P_{13} - P_{23} - P_{25}).\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{35} &= \frac{4}{P_{25}} \sum_{a=1}^2 \sum_{b=1}^2 \frac{|\mathcal{V}_{a1}|^2 |\mathcal{V}_{b1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{23})} [2U_{35} - m_b^2 W_{35}] , \\ \mathbf{U}_{35} &= P_{23} \left\{ m_{\tilde{\nu}}^2 (P_{12} - P_{15}) - 2P_{14} (P_{24} - P_{45}) \right\} , \\ \mathbf{W}_{35} &= -P_{12} P_{45} - P_{14} P_{25} + P_{15} P_{24}.\end{aligned}$$

$$\mathbf{T}_{44} = \frac{4}{P_{15}} \sum_{a=1}^2 \sum_{b=1}^2 \frac{|\mathcal{V}_{a1}|^2 |\mathcal{V}_{b1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{23})} [2P_{23} P_{35} - m_{\tilde{\nu}}^2 P_{25}] .$$

$$\mathbf{T}_{45} = \frac{4}{P_{15}} \sum_{a=1}^2 \sum_{b=1}^2 \frac{|\mathcal{V}_{a1}|^2 |\mathcal{V}_{b1}|^2}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{23})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{14})} [2U_{45} - m_b^2 W_{45}]$$

$$\mathbf{U}_{45} = m_{\tilde{\nu}}^2 \left\{ P_{12}(P_{12} - P_{13} - P_{15} - P_{25}) + P_{25}(P_{13} + P_{15}) \right\} - 2P_{13}P_{23}(P_{12} - P_{13} - P_{15} + P_{35}) + 2P_{23}P_{35}(P_{12} - P_{15}),$$

$$\mathbf{W}_{45} = -P_{12}P_{35} + P_{13}P_{25} - P_{15}P_{23}.$$

$$\mathbf{T}_{55} = \frac{8}{P_{15}} \sum_{a=1}^2 \sum_{b=1}^2 \frac{|\mathcal{V}_{a1}|^2 |\mathcal{V}_{b1}|^2 [U_{55} - 2(m_a^2 + m_b^2)W_{55} + m_a^2 m_b^2 P_{12}]}{(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_a^2 - 2P_{23})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{14})(m_{\tilde{\nu}}^2 - m_b^2 - 2P_{23})},$$

$$\mathbf{U}_{55} = m_{\tilde{\nu}}^2 \left\{ P_{12}(m_{\tilde{\nu}}^2 - 4P_{23} + 2P_{35}) - 2P_{13}P_{25} + 2P_{15}(P_{23} - P_{25}) \right\} + 4P_{23}(P_{23} - P_{35})(P_{12} - P_{15}) + 4P_{13}P_{23}P_{25},$$

$$\mathbf{W}_{55} = P_{23}(-P_{12} + P_{13} + P_{15}).$$

$$T(\tilde{\nu}\tilde{\nu}) = \sum_{\substack{i,j=1 \\ j \geq 1}}^5 T_{ij}$$

C : The process $e^+ e^- \longrightarrow \widetilde{N}_i \widetilde{N}_j + \gamma$.

In this subsection we give the formula for the cross section of the process $e^+ e^- \longrightarrow \widetilde{N}_i \widetilde{N}_j + \gamma$. The contributing Feynman diagrams are as in Fig.[13]. We define for this subsection:

$$S = m_i^2 + m_j^2 + 2P_{34} - M_Z^2$$

$$B_W = \frac{1}{(m_i^2 + m_j^2 + 2P_{34} - M_Z^2)^2 + M_Z^2 G_Z^2}$$

In this subsection we label the particles by the following indices: 3 and 4 stand for \widetilde{N}_i and \widetilde{N}_j respectively where $i, j=1,2$ and $i \leq j$. The labels 1,2,5 have the same meaning as in the previous subsection.

The convention for the T_{ij} -s in this subsection is the same as in the last subsection for the process under consideration.

$$\mathbf{T}_{11} = -\frac{(C_A^2 + C_V^2)G_A^2}{2P_{25}}[m_i m_j P_{15} - P_{14}P_{35} - P_{13}P_{45}]$$

$$\mathbf{T}_{12} = -\frac{(C_A^2 + C_V^2)G_A^2}{2P_{15}P_{25}}[(P_{12} - P_{15} - P_{25})(2m_i m_j P_{12} - P_{13}P_{24} - P_{14}P_{23}) - P_{12}\{P_{13}P_{24} + P_{14}P_{23} - P_{35}(P_{14} + P_{24}) - P_{45}(P_{13} + P_{23})\} - 2(P_{15}P_{23}P_{24} + P_{13}P_{14}P_{25})]$$

$$\mathbf{T}_{13} = -\frac{2C_L G_A}{P_{25}(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)}[m_i m_j P_{15} - 2P_{14}P_{35}]$$

$$\mathbf{T}_{14} = -\mathbf{T}_{13} \begin{pmatrix} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{pmatrix}$$

$$\mathbf{T}_{15} = \frac{2C_L G_A}{P_{15}P_{25}(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)}[(P_{12} - P_{15} - P_{25})(m_i m_j P_{12} - P_{14}P_{23}) - P_{23}(P_{12}P_{14} + P_{15}P_{24} - P_{12}P_{45}) - P_{14}(P_{13}P_{25} - P_{12}P_{35})]$$

$$\mathbf{T}_{16} = -\mathbf{T}_{15} \begin{pmatrix} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{pmatrix}$$

$$\mathbf{T}_{17} = \frac{C_L G_A}{P_{25}(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)}[m_i m_j \{P_{12}(-2P_{12} + 2P_{15} - 2P_{23} + 2P_{24} + P_{35} - P_{45})$$

$$+ P_{15}(P_{23} - P_{24}) - P_{25}(P_{13} - P_{14})\} + 2P_{14}\{P_{23}(2P_{12} - P_{15} + 2P_{23}$$

$$- 2P_{24} - 2P_{35} + P_{45}) + P_{25}(m_i^2 + P_{13} - P_{34}) - P_{35}(P_{12} - P_{24})\}]]$$

$$\mathbf{T}_{18} = -\mathbf{T}_{17} \begin{pmatrix} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{pmatrix}$$

$$\mathbf{T}_{22} = \mathbf{T}_{11}(1 \leftrightarrow 2)$$

$$\mathbf{T}_{23} = \frac{2C_L G_A}{P_{15}P_{25}(m_i^2 - 2P_{14} - m_{\tilde{e}_R}^2)}[(P_{12} - P_{15} - P_{25})(m_i m_j P_{12} - P_{14}P_{23}) - P_{23}(P_{12}P_{14} - P_{12}P_{45} + P_{15}P_{24}) - P_{14}(P_{13}P_{25} - P_{12}P_{35})]$$

$$\mathbf{T}_{24} = -\mathbf{T}_{23} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{25} = \frac{2C_L G_A}{P_{15}(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)} [m_i m_j P_{25} - 2P_{23} P_{45}]$$

$$\mathbf{T}_{26} = -\mathbf{T}_{25} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{27} = \frac{C_L G_A}{P_{15}(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)} [m_i m_j \{ P_{12}(-2P_{12} + 2P_{13} - 2P_{14} +$$

$$+ P_{15}(P_{23} - 2P_{24}) - P_{25}(P_{13} - P_{14}) \} + 2P_{23} \{ P_{14}(2P_{12} - 2P_{13} + 2P_{14}$$

$$- P_{25} + P_{35} - 2P_{45}) + P_{15}(m_j^2 + P_{24} - P_{34}) - P_{45}(P_{12} - P_{13}) \}]$$

$$\mathbf{T}_{28} = -\mathbf{T}_{27} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{33} = \frac{16P_{14}P_{35}}{P_{25}(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)^2}$$

$$\mathbf{T}_{34} = \frac{16m_i m_j P_{15}}{P_{25}(m_i^2 - 2P_{13} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)}$$

$$\mathbf{T}_{35} = \frac{16}{P_{15}P_{25}(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)} [P_{14}P_{23}(2P_{12} - P_{15} - P_{25}) +$$

$$P_{14}(P_{13}P_{25} - P_{12}P_{35})$$

$$+ P_{23}(P_{15}P_{24} - P_{12}P_{45})]$$

$$\mathbf{T}_{36} = \frac{16m_i m_j P_{12}(P_{12} - P_{15} - P_{25})}{P_{15}P_{25}(m_j^2 - 2P_{24} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)}$$

$$\mathbf{T}_{37} =$$

$$\frac{-16P_{14}}{P_{25}(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)^2} [P_{23}(2P_{12} - P_{15} + 2P_{23} - 2P_{24} - 2P_{35} + P_{45})$$

$$+P_{25}(m_i^2 + P_{13} - P_{34}) - P_{35}(P_{12} - P_{24})\Big]$$

$$\mathbf{T}_{38} = \frac{-8m_i m_j}{P_{25}(m_i^2 - 2P_{13} - m_{e_R}^2)(m_j^2 - 2P_{14} - m_{e_R}^2)(m_j^2 - 2P_{24} - m_{e_R}^2)} \Big[P_{12}(2P_{12} - 2P_{15} -$$

$$+P_{35} - P_{45}) + P_{15}(P_{23} - P_{24}) - P_{25}(P_{13} - P_{14})\Big]$$

$$\mathbf{T}_{44} = \mathbf{T}_{33} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{45} = \mathbf{T}_{36} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{46} = \mathbf{T}_{35} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{47} = \mathbf{T}_{38} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{48} = \mathbf{T}_{37} \left\{ \begin{array}{l} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{55} = \frac{16P_{23}P_{45}}{P_{15}(m_i^2 - 2P_{23} - m_{e_R}^2)^2}$$

$$\mathbf{T}_{56} = \frac{16m_i m_j P_{25}}{P_{15}(m_i^2 - 2P_{23} - m_{e_R}^2)(m_j^2 - 2P_{24} - m_{e_R}^2)}$$

$$\mathbf{T}_{57} = \frac{-16P_{23}}{P_{15}(m_i^2 - 2P_{23} - m_{e_R}^2)^2(m_j^2 - 2P_{14} - m_{e_R}^2)} \Big[(P_{12} - P_{13} + P_{14})(2P_{14} - P_{45})$$

$$-P_{14}(P_{25} - P_{35} + P_{45}) + P_{15}(m_j^2 + P_{24} - P_{34})\Big]$$

$$\mathbf{T}_{58} = \frac{-8m_i m_j}{P_{15}(m_i^2 - 2P_{13} - m_{\tilde{e}_R}^2)(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)(m_j^2 - 2P_{24} - m_{\tilde{e}_R}^2)} \left[(P_{13} - P_{14})(2P_{12} - P_{25}) \right.$$

$$\left. + P_{12} \{ 2(P_{12} - P_{25}) - (P_{35} - P_{45}) \} + P_{15}(P_{23} - P_{24}) \right]$$

$$\mathbf{T}_{66} = \mathbf{T}_{55} \left\{ \begin{array}{c} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{67} = \mathbf{T}_{58} \left\{ \begin{array}{c} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{68} = \mathbf{T}_{57} \left\{ \begin{array}{c} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{77} = \frac{-16P_{14}P_{23}}{(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2)^2 (m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2)^2} \left[m_i^2 + m_j^2 - 2(P_{12} - P_{13} + P_{14} + P_{23} - P_{24} + P_{34}) \right]$$

$$\mathbf{T}_{78} = \frac{16m_i m_j P_{12} [m_i^2 + m_j^2 + 2(P_{12} - P_{34})]}{(m_i^2 - 2P_{23} - m_{\tilde{e}_R}^2) (m_i^2 - 2P_{13} - m_{\tilde{e}_R}^2) (m_j^2 - 2P_{14} - m_{\tilde{e}_R}^2) (m_j^2 - 2P_{24} - m_{\tilde{e}_R}^2)}$$

$$\mathbf{T}_{88} = \mathbf{T}_{77} \left\{ \begin{array}{c} i \leftrightarrow j \\ 3 \leftrightarrow 4 \end{array} \right\}$$

$$\mathbf{T}_{st} = T_{13} - T_{14} + T_{15} - T_{16} + T_{17} - T_{18}$$

$$+ T_{23} - T_{24} + T_{25} - T_{26} + T_{27} - T_{28}$$

$$\mathbf{T}_{t1} = T_{33} + T_{44} + T_{55} + T_{66} + T_{77} + T_{88} + T_{35} + T_{37} + T_{46} + T_{48} + T_{57} + T_{68}$$

$$\mathbf{T}_{t2} = T_{34} + T_{36} + T_{38} + T_{45} + T_{47} + T_{56} + T_{58} + T_{67} + T_{78}$$

$$\begin{aligned} T(\widetilde{N}_i \widetilde{N}_j) &= B_W(T_{11} + T_{12} + T_{22}) \\ &+ \sum_{h=L,R} \left[B_W S \{ (a_i)_h (a_j)_h T_{st} \} + \{ (a_i)_h (a_j)_h \}^2 T_{t1} + C_{ij} S_{ij} \{ (a_i)_h (a_j)_h \}^2 T_{t2} \right] \end{aligned}$$

where

$$\begin{aligned}
G_A &= N'_{i3}N'_{j3} - N'_{i4}N'_{j4} \\
(a_i)_L &= (0.5 + S_W^2)N'_{i2} - S_W C_W N'_{i1} \\
(a_i)_R &= -S_W^2 N'_{i2} + S_W C_W N'_{i1} \\
(a_j)_L &= (0.5 + S_W^2)N'_{j2} - S_W C_W N'_{j1} \\
(a_j)_R &= -S_W^2 N'_{j2} + S_W C_W N'_{j1}
\end{aligned}$$

where the matrix N' diagonalises the 4×4 neutralino mass matrix following the convention of Haber and Kane[1].

The Chiral Rotation Factor (C_{ij}) and the Fermi statistics Factor (S_{ij}) are defined as follows:

$$\begin{aligned}
C_{ij} &= +1 \quad \text{for } m_{\tilde{N}_1}, m_{\tilde{N}_2} > 0 \\
&= -1 \quad \text{for either } m_{\tilde{N}_1} \text{ or } m_{\tilde{N}_2} < 0 \\
S_{ij} &= +1 \quad \text{for } i \neq j \\
&= -1 \quad \text{for } i = j
\end{aligned}$$

Table Captions

Table-I :

The comparison of the response of the signal to two sets of cuts **A**[6] and **B**[15] at $\sqrt{s}=190$ GeV where Cut **A** $\equiv 5 < E_\gamma < 60$ GeV, $40^\circ < \theta_\gamma < 140^\circ$; Cut **B** $\equiv 1 < E_\gamma < 47.5$ GeV, $18^\circ < \theta_\gamma < 162^\circ$, $p_{T_\gamma} > 6.175$ GeV. Other fixed values of the SUSY parameters used are $(\mu, m_{\tilde{g}}, \tan\beta) = (-300 \text{ GeV}, 200 \text{ GeV}, 10)$ and $m_{\tilde{e}_L} = m_{\tilde{e}_R}$. The SM background with Cut **A**(**B**) is 0.45(0.51)pb. All masses are in GeV and cross-sections in picobarns.

Table-II :

The comparison of the response of the signal to two sets of cuts **A** and **B**[18] at $\sqrt{s}=350$ GeV where Cut **A** $\equiv 65 < E_\gamma < 150$ GeV, $40^\circ < \theta_\gamma < 140^\circ$; Cut **B** $\equiv 1 < E_\gamma < 150$ GeV, $10^\circ < \theta_\gamma < 170^\circ$, $p_{T_\gamma} > 10$ GeV. Other fixed values of the SUSY parameters used are $(\mu, m_{\tilde{g}}, \tan\beta) = (-500 \text{ GeV}, 400 \text{ GeV}, 2)$ and $m_{\tilde{e}_L} = m_{\tilde{e}_R}$. The SM background with Cut **A**(**B**) is 0.07671(1.04496)pb. All masses are in GeV and cross-sections in picobarns.

Table-III :

The comparison of the response of the signal to two sets of cuts **A** and **B**[18] at $\sqrt{s}=500$ GeV where Cut **A** $\equiv 95 < E_\gamma < 225$ GeV, $40^\circ < \theta_\gamma < 140^\circ$; Cut **B** $\equiv 1 < E_\gamma < 225$ GeV, $10^\circ < \theta_\gamma < 170^\circ$, $p_{T_\gamma} > 10$ GeV. Other fixed values of the SUSY parameters used are $(\mu, m_{\tilde{g}}, \tan\beta) = (-500 \text{ GeV}, 450 \text{ GeV}, 2)$ and $m_{\tilde{e}_L} = m_{\tilde{e}_R}$. The SM background with Cut **A**(**B**) is 0.07227(1.48773)pb. All masses are in GeV and cross-sections in picobarns.

Table-IV :

Total signal cross section at $\sqrt{s} = 190$ and 350 GeV in $N = 1$ SUGRA model using Cut **A** of Tables I and II. The SUSY parameters consistent with the VLSP scenario are chosen from [7]. The underlined entries correspond to the representative choice $\mu = -m_{\tilde{g}}$ leading to radiative breaking of $SU(2) \otimes U(1)$ symmetry. All masses are in GeV and cross-sections in picobarns.

Table-V :

Effect of soft photon radiative correction at LEP-2 energies. For each $m_{\tilde{\nu}}$ the cross sections with and without radiative corrections are presented using the cut **A** of Table-I. The choice of SUSY parameters is $(\mu, m_{\tilde{g}}, \tan\beta)=(-300 \text{ GeV}, 200 \text{ GeV}, 10)$. All masses are in GeV and cross-sections in picobarns.

Figure Captions

Fig. 1 The region in the $m_{\tilde{\nu}} - m_{\tilde{g}}$ plane compatible with the VLSP scenario with $\tan\beta = 10$ for **(A)** $\mu = -250$ and **(B)** 250 GeV (see section 2 for further explanations).

Fig. 2 photon at $\sqrt{s} = 190$ GeV, with $E_{\gamma} > 5$ GeV and $5^{\circ} < \theta_{\gamma} < 175^{\circ}$. **(B)** Angular distribution of the photon at $\sqrt{s} = 190$ GeV with $5 < E_{\gamma} < 60$ GeV and $5^{\circ} < \theta_{\gamma} < 175^{\circ}$. The convention for different lines and SUSY parameters chosen are explained in section 2.

Fig. 3 The total cross section (SM+VLSP) as a function of $m_{\tilde{\nu}}$ at $\sqrt{s} = 190$ GeV. The band within the solid lines correspond to the SM+ $\tilde{\nu}\tilde{\nu} + \tilde{N}_1\tilde{N}_1$ cross section and is obtained by varying $m_{\tilde{g}}$, μ and $\tan\beta$ over the LEP-1 allowed region. The band within the dashed lines is obtained by taking additional contributions from $\tilde{N}_1\tilde{N}_2$ pairs into account. The horizontal dotted lines correspond to the SM background and its fluctuations (see section 2).

Fig. 4 Contour plots in the $(m_{\tilde{\nu}} - m_{\tilde{g}})$ plane at $\sqrt{s} = 190$ GeV indicating the regions where $\geq 2\sigma$ (dotted), $\geq 3\sigma$ (dashed) and $\geq 4\sigma$ (solid) signals may be obtained for $\tan\beta = 2, 10$ and 30 and $-500 \leq \mu \leq 500$. The shaded region corresponds to $m_{\tilde{\nu}} < M_{\tilde{\chi}_1^{\pm}} < m_{\tilde{\nu}} + 5$ GeV, where the chargino decay is likely to be difficult to detect.

Fig. 5 **(A)** Energy distribution of the photon at $\sqrt{s} = 350$ GeV, with $E_{\gamma} > 5$ GeV and $5^{\circ} < \theta_{\gamma} < 175^{\circ}$. **(B)** Angular distribution for the photon at $\sqrt{s} = 350$ GeV with $15 < E_{\gamma} < 150$ and $5^{\circ} < \theta_{\gamma} < 175^{\circ}$. The convention for different lines and SUSY parameters chosen are explained in section 3.

Fig. 6 The total cross section (SM+VLSP) as a function of $m_{\tilde{\nu}}$ at $\sqrt{s} = 350$ GeV. The conventions for different bands and lines are explained in Fig.[3] and in section 3.

Fig. 7 Contour plots in the $(m_{\tilde{\nu}} - m_{\tilde{g}})$ plane at $\sqrt{s} = 350$ GeV indicating the regions where $\geq 3\sigma$ (dotted), $\geq 4\sigma$ (dashed) and $\geq 5\sigma$ (solid) signals may be obtained for $\tan\beta = 2, 10$ and 30 and $-500 \leq \mu \leq 500$.

Fig. 8 **(A)** Energy distribution of the photon at $\sqrt{s} = 500$ GeV, with $E_\gamma > 5$ GeV and $5^\circ < \theta_\gamma < 175^\circ$. **(B)** Angular distribution for the photon at $\sqrt{s} = 500$ GeV with $25 < E_\gamma < 225$ and $5^\circ < \theta_\gamma < 175^\circ$. The convention for different lines and SUSY parameters chosen are explained in the text.

Fig. 9 The total cross section (SM+VLSP) as a function of $m_{\tilde{\nu}}$ at $\sqrt{s} = 500$ GeV. The conventions are the same as in Fig.[6].

Fig. 10 Contour plots in the $(m_{\tilde{\nu}} - m_{\tilde{g}})$ plane at $\sqrt{s} = 500$ GeV following the conventions of Fig.[7].

Fig. 11 Feynman diagrams for the process $e^+ e^- \longrightarrow \nu \bar{\nu} + \gamma$. In this and in the subsequent figures, the index l stands for e, μ, τ . Arrows indicate the flow of physical momenta and not of fermion numbers.

Fig. 12 Feynman diagrams for the process $e^+ e^- \longrightarrow \tilde{\nu} \tilde{\nu} + \gamma$.

Fig. 13 Feynman diagrams for the process $e^+ e^- \longrightarrow \widetilde{N}_i \widetilde{N}_j + \gamma$.

Table-I

$m_{\tilde{\nu}}$	Cut	$\sigma_{\tilde{\nu}\tilde{\nu}}$	$\sigma_{\tilde{N}_1\tilde{N}_1}$	$\sigma_{\tilde{N}_1\tilde{N}_2}$	Total	$\sigma = \frac{S}{\sqrt{B}}$
45	A	.103	.027	.018	.148	4.9
	B	.117	.031	.022	.170	5.3
55	A	.077	.025	.016	.118	3.9
	B	.087	.029	.019	.135	4.2
65	A	.048	.023	.015	.086	2.8
	B	.053	.026	.017	.096	3.0

Table-II

$m_{\tilde{\nu}}$	Cut	$\sigma_{\tilde{\nu}\tilde{\nu}}$	$\sigma_{\tilde{N}_1\tilde{N}_1}$	$\sigma_{\tilde{N}_1\tilde{N}_2}$	$\sigma_{\tilde{N}_2\tilde{N}_2}$	Total	$\sigma = \frac{S}{\sqrt{B}}$
110	A	.00156	.00212	disallowed [!]	disallowed [!]	.00368	2.3
	B	.02037	.01803			.03840	6.7
125	A	.00041	.00182	disallowed [!]	disallowed [!]	.00223	1.4
	B	.01194	.01600			.02794	4.8
130	A	.00018	.00173	.00143	negligible	.00334	2.2
	B	.00948	.01539	.01397	.00101	.03985	6.9
135	A	.00004	.00165	.00134	negligible	.00303	1.9
	B	.00721	.01476	.01324	.00096	.03617	6.3
*150	A	negligible	.00113	.00074	negligible	.00187	1.2
	B	.00173	.01162	.01060	.00013	.02408	4.1

* In this case $m_{\tilde{g}}=450$ GeV, since $m_{\tilde{g}} = 400$ GeV is not allowed in the VLSP scenario.

[!] \tilde{N}_2 cannot be a VLSP for this choice of SUSY parameters.

Table-III

$m_{\tilde{\nu}}$	Cut	$\sigma_{\tilde{\nu}\tilde{\nu}}$	$\sigma_{\tilde{N}_1\tilde{N}_1}$	$\sigma_{\tilde{N}_1\tilde{N}_2}$	$\sigma_{\tilde{N}_2\tilde{N}_2}$	Total	$\sigma = \frac{S}{\sqrt{B}}$
150	A	.00138	.00156	.00130	.00012	.00436	2.9
	B	.02207	.01417	.01114	.00344	.05082	7.4
155	A	.00113	.00150	.00124	.00012	.00399	2.6
	B	.02030	.01378	.01073	.00333	.04814	7.0
[†] 160	A	.00078	.00129	disallowed [!]	disallowed [!]	.00207	1.4
	B	.01582	.01254			.02836	4.1
[†] 165	A	.00060	.00124	.00106	.00004	.00294	1.9
	B	.01430	.01218	.01013	.00231	.03892	5.7
[†] 170	A	.00044	.00120	.00101	.00003	.00268	1.8
	B	.01287	.01185	.00975	.00223	.03670	5.3
[‡] 180	A	.00017	.00098	.00083	negligible	.00198	1.3
	B	.00852	.01048	.00918	.00136	.02954	4.3

[†] $m_{\tilde{g}}=500$ GeV

[!] \tilde{N}_2 cannot be a VLSP for this choice of SUSY parameters.

[‡] $m_{\tilde{g}}=550$ GeV.

Table-IV

\sqrt{s}	$\tan\beta$	m_0	M_2	μ	$m_{\tilde{g}}(m_{\tilde{q}})$	$m_{\tilde{\nu}}$	σ_{SUSY}	$\sigma = \frac{S}{\sqrt{B}}$
190	2	20	75	-55	225(225)	47.5	.141	4.7
		40	70	-65	210(213)	53.8	.108	3.6
		30	70	320	210(211)	46.8	.219	7.3
		40	70	400	210(213)	53.8	.170	5.6
		50	70	700	210(215)	61.6	.124	4.1
		50	75	800	225(230)	66.0	.099	3.3
350	2	20	75	-55	225(225)	47.5	.03478	22.3
		40	80	-75	240(242)	63.7	.02338	15.0
		75	70	-125	210(222)	83.2	.01838	11.8
		30	70	320	210(211)	46.8	.05099	32.8
		40	70	500	210(213)	53.8	.04164	26.8
		50	70	1000	210(215)	61.6	.03467	22.3
		55	80	1000	240(245)	74.0	.02494	16.0
190	10	60	70	-160	210(218)	57.6	.121	4.1
		60	70	<u>-210</u>	<u>210(218)</u>	57.6	<u>.118</u>	3.9
		40	80	-120	240(242)	49.7	.144	4.8
		40	90	-120	270(272)	61.5	.094	3.1
		20	90	120	270(270)	50.9	.155	5.1
		60	70	280	210(218)	57.6	.128	4.2
		60	70	500	210(218)	57.6	.121	4.0
350	10	60	70	<u>-210</u>	<u>210(218)</u>	57.6	<u>.03388</u>	21.8
		60	80	-60	240(246)	66.9	.02648	17.0
		60	80	<u>-240</u>	<u>240(246)</u>	66.9	<u>.02605</u>	16.7
		60	90	-200	270(276)	76.1	.02014	12.9
		75	100	<u>-300</u>	<u>300(308)</u>	96.3	<u>.01169</u>	7.5
		40	90	-120	270(272)	61.5	.02789	17.9
		40	90	160	270(272)	61.5	.02990	19.2
		60	100	280	300(305)	85.2	.01292	8.3
		70	85	500	255(263)	80.1	.01960	12.6

Table-V

$m_{\tilde{\nu}}$	$\sigma_{\tilde{\nu}\tilde{\nu}}$	$\sigma_{\tilde{N}_1\tilde{N}_1}$	$\sigma_{\tilde{N}_1\tilde{N}_2}$	Total	$\sigma = \frac{S}{\sqrt{B}}$
45	.103	.027	.018	.148	4.9
	.089	.020	.014	.123	4.7
55	.077	.025	.016	.118	3.9
	.057	.019	.013	.089	3.4
65	.048	.023	.015	.086	2.8
	.034	.017	.011	.062	2.4

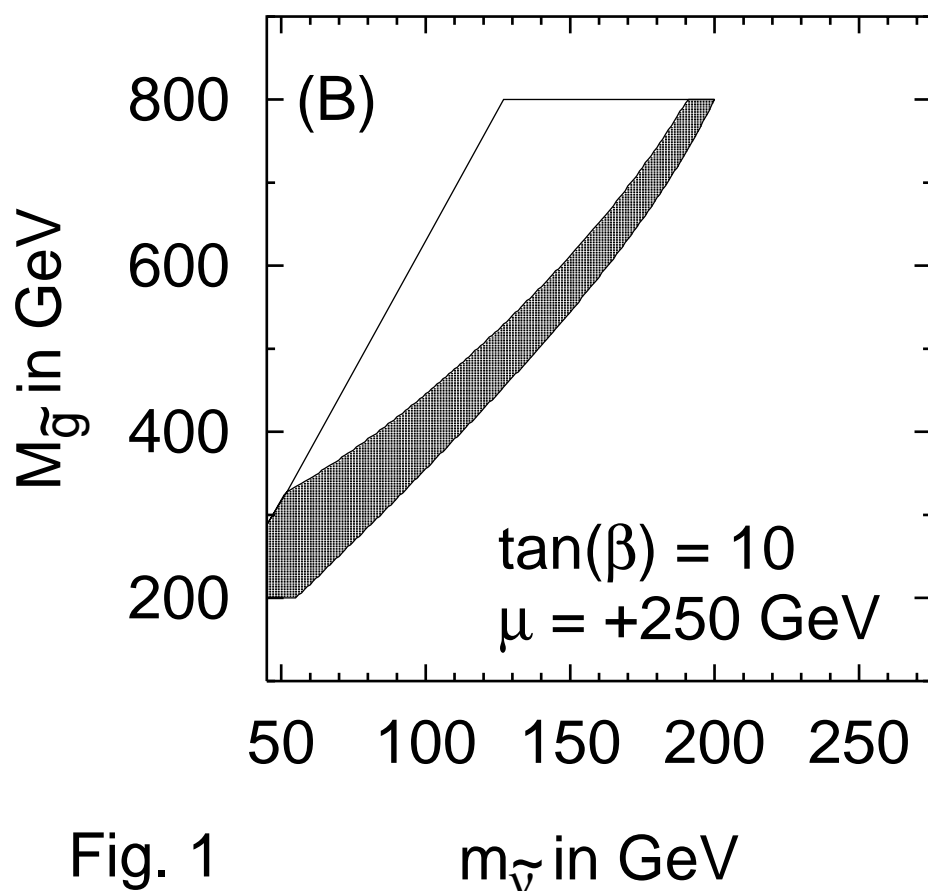
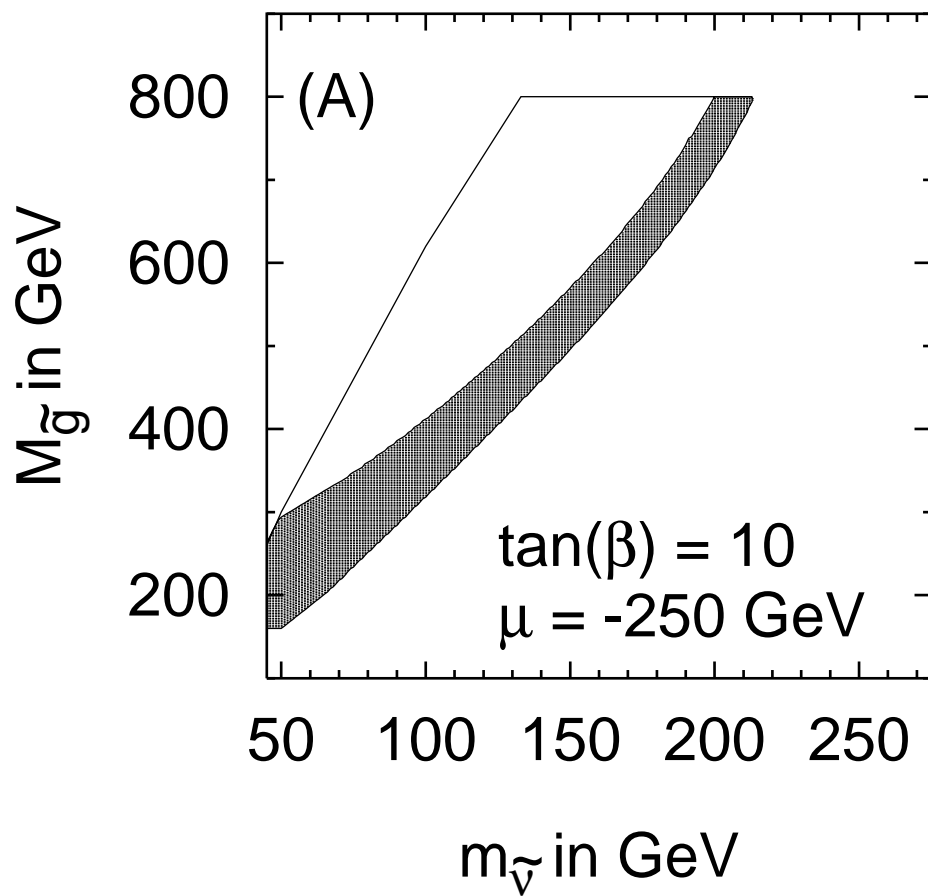


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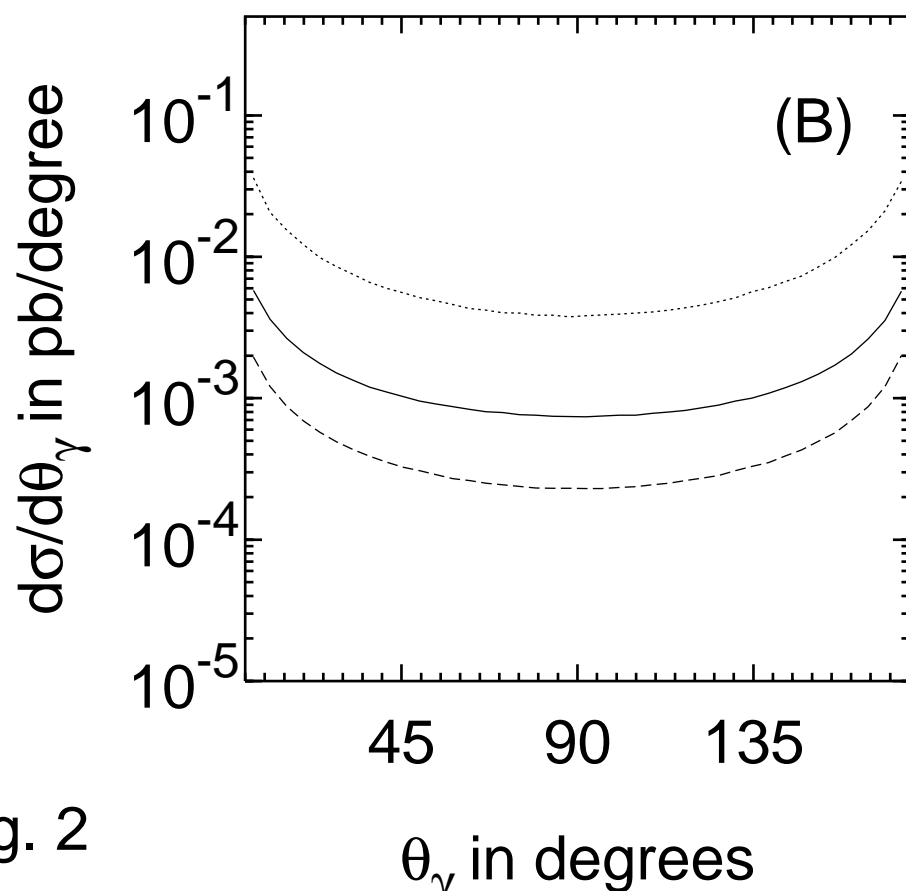
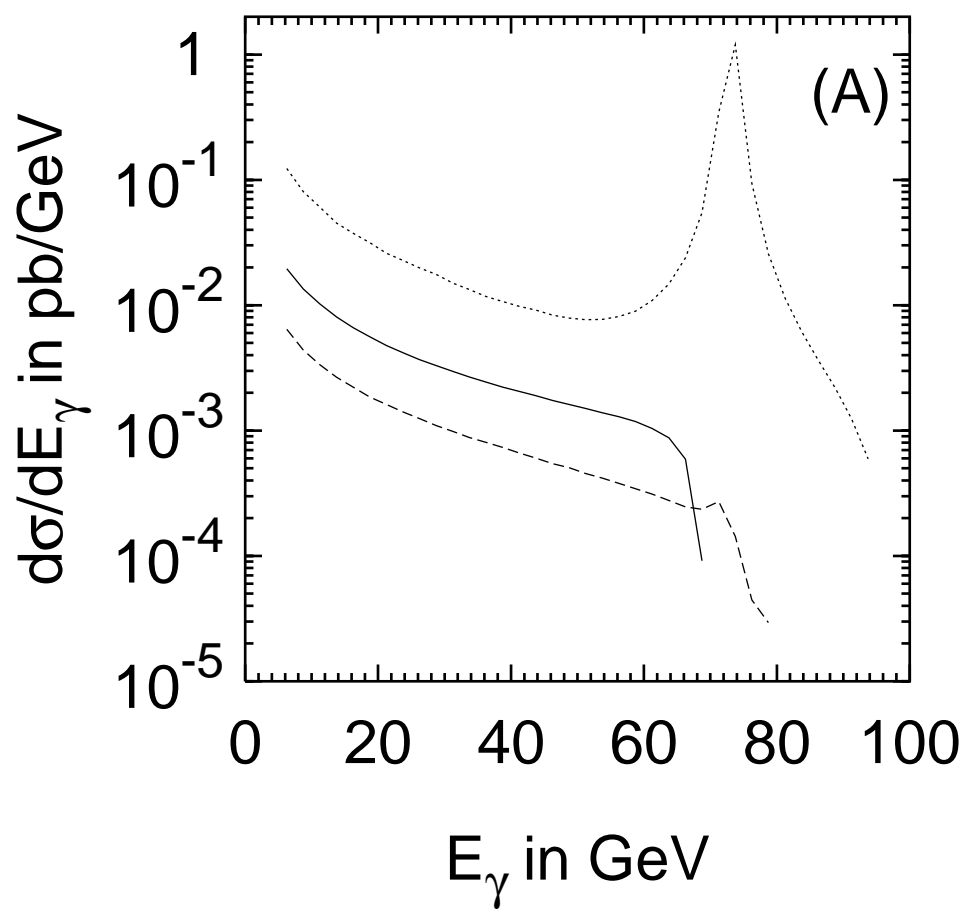


Fig. 2

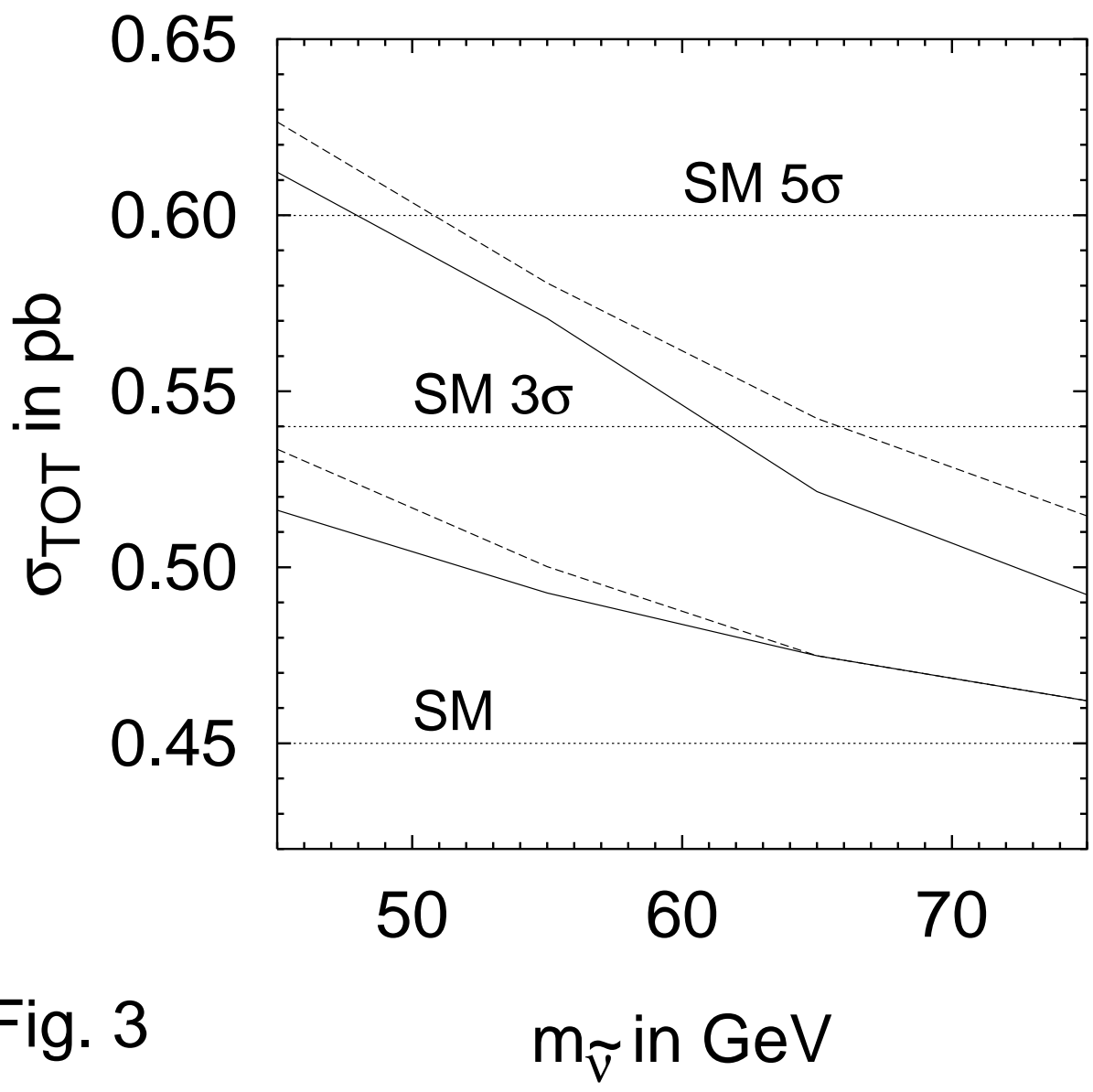
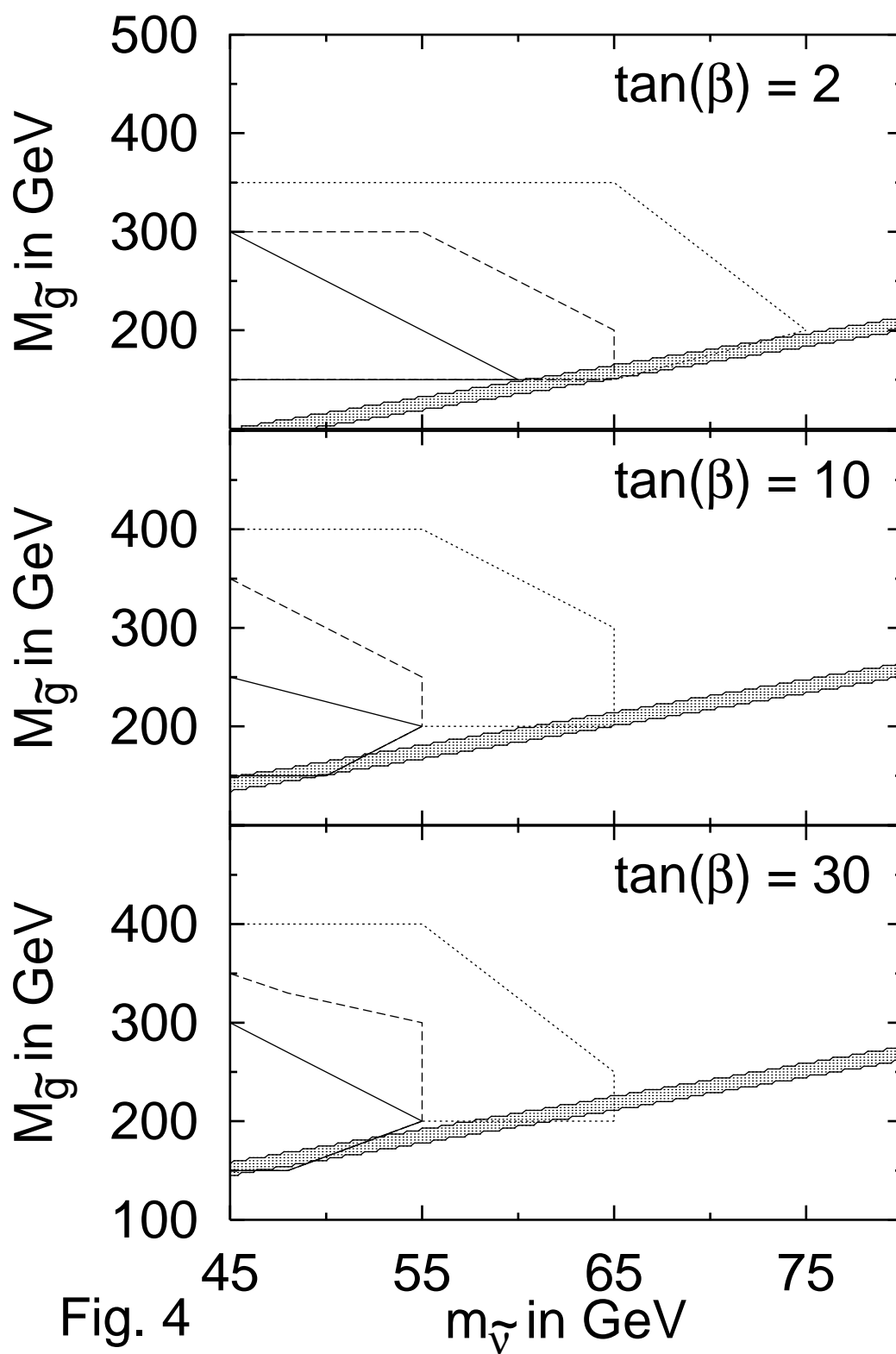


Fig. 3



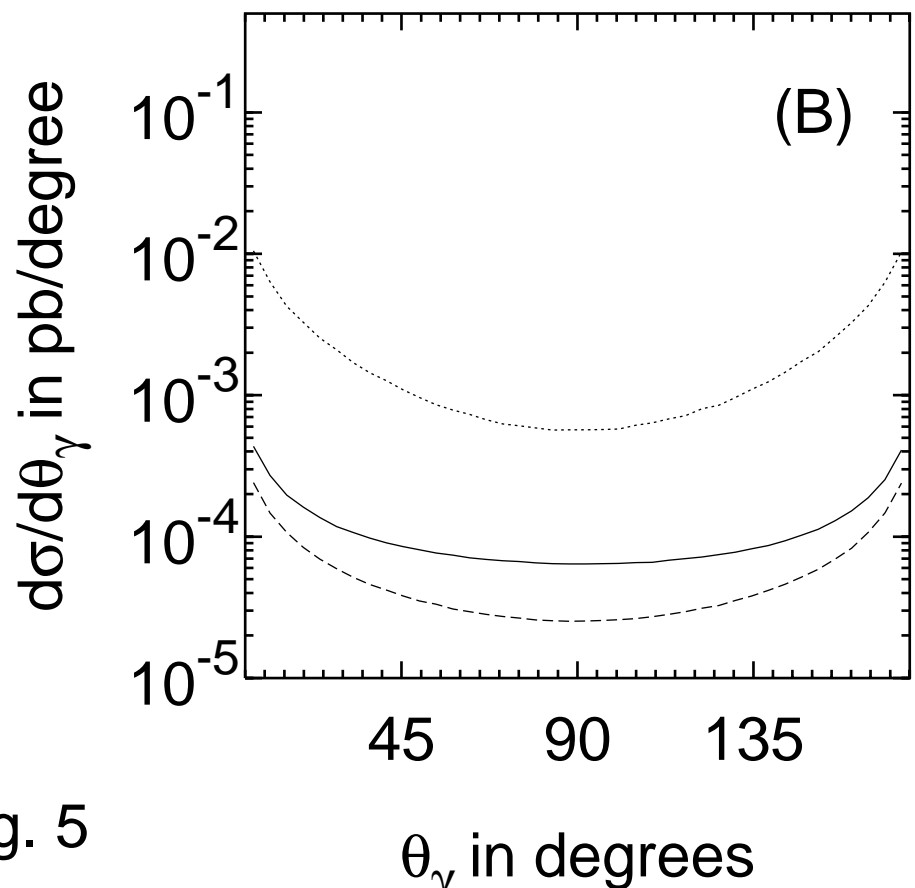
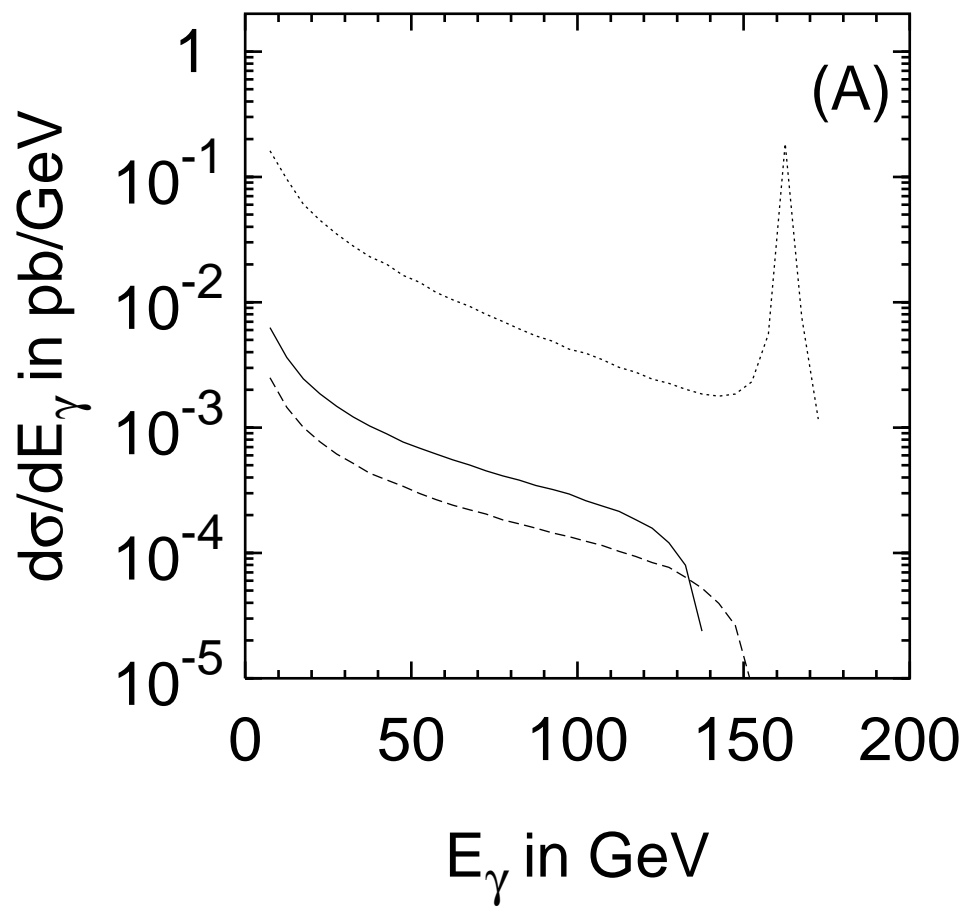


Fig. 5

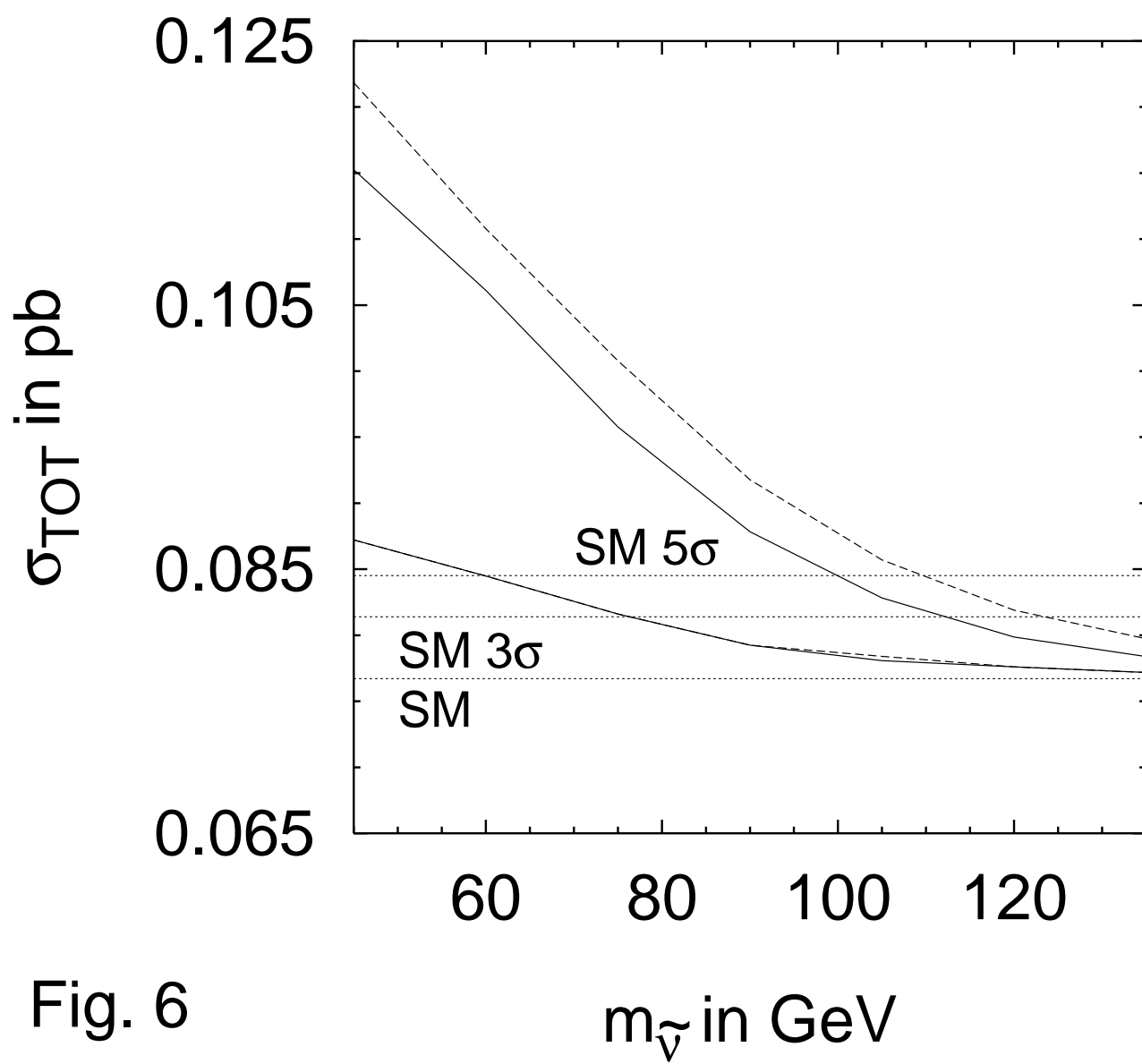


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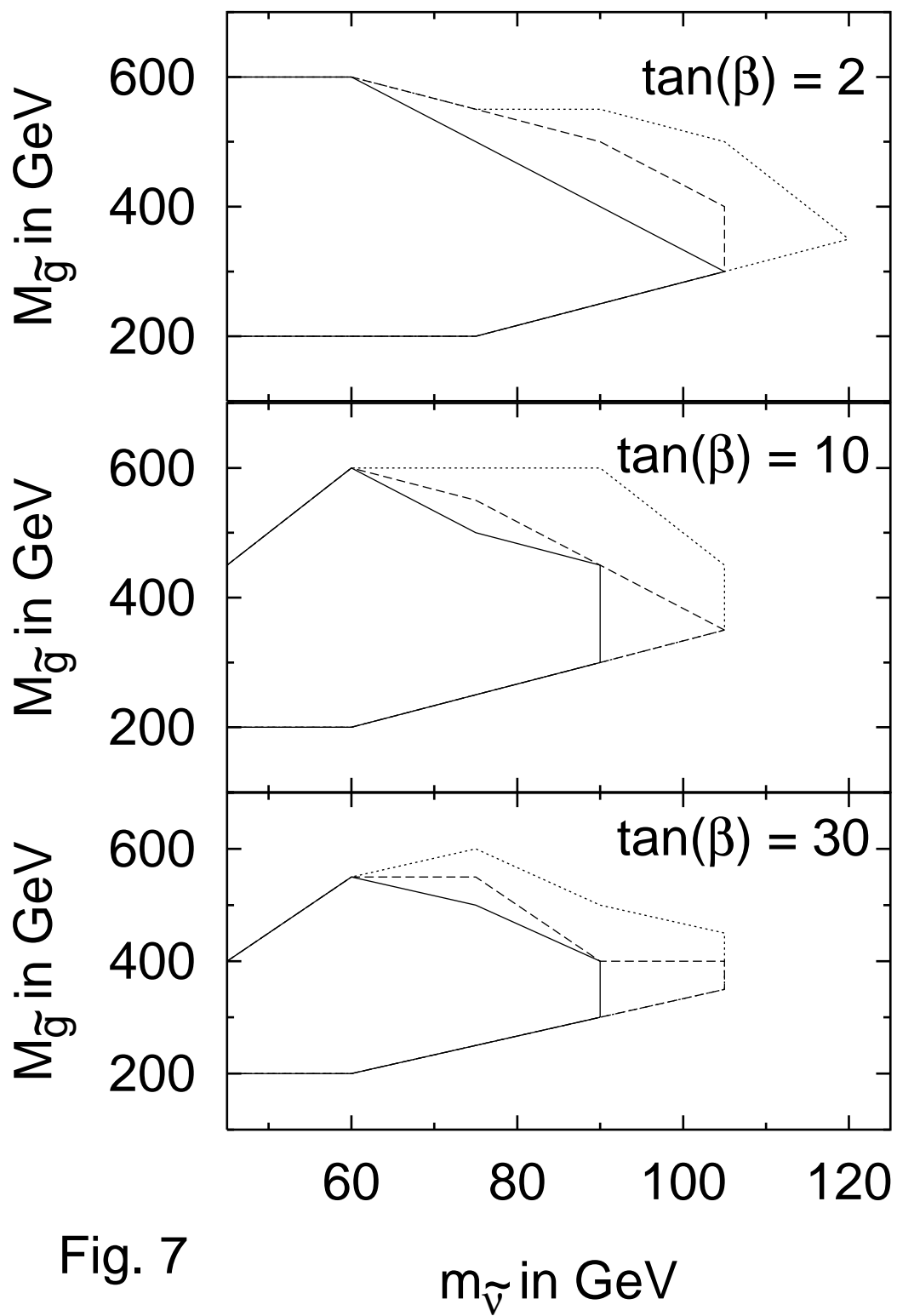


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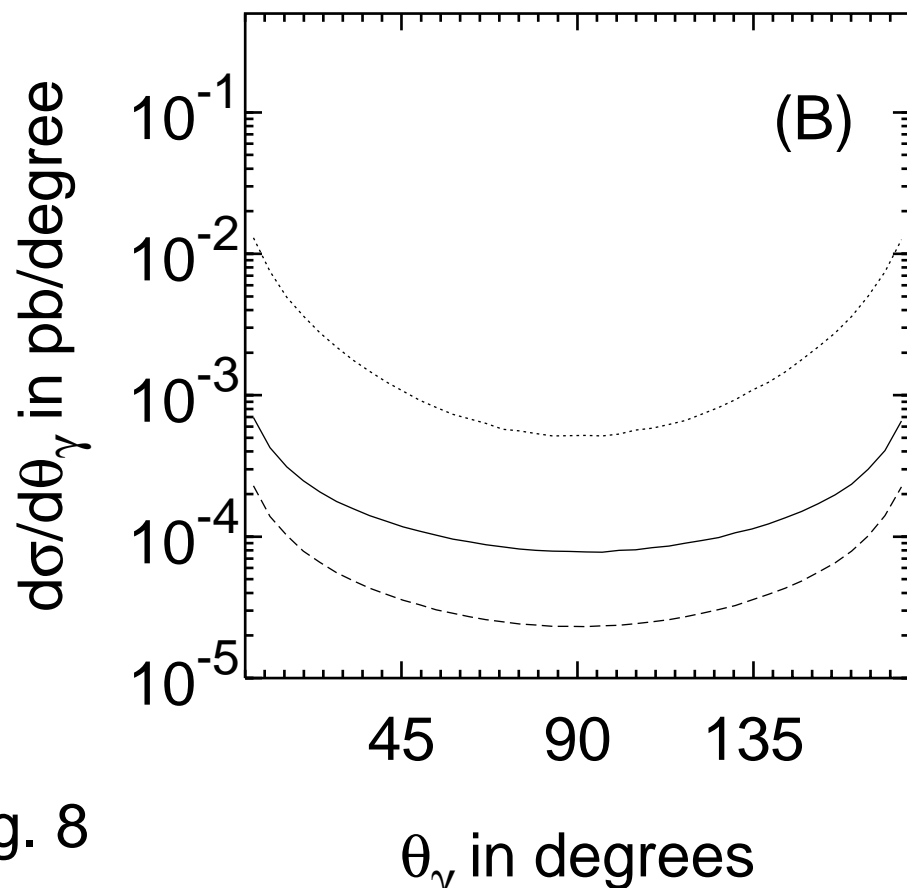
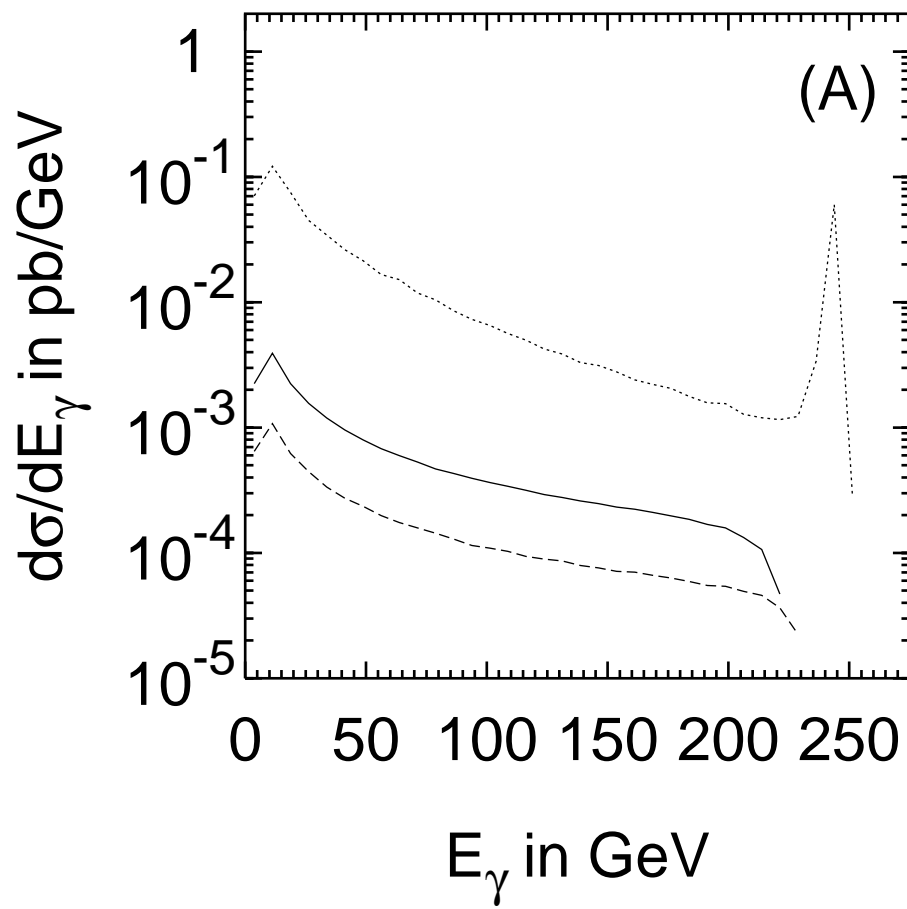


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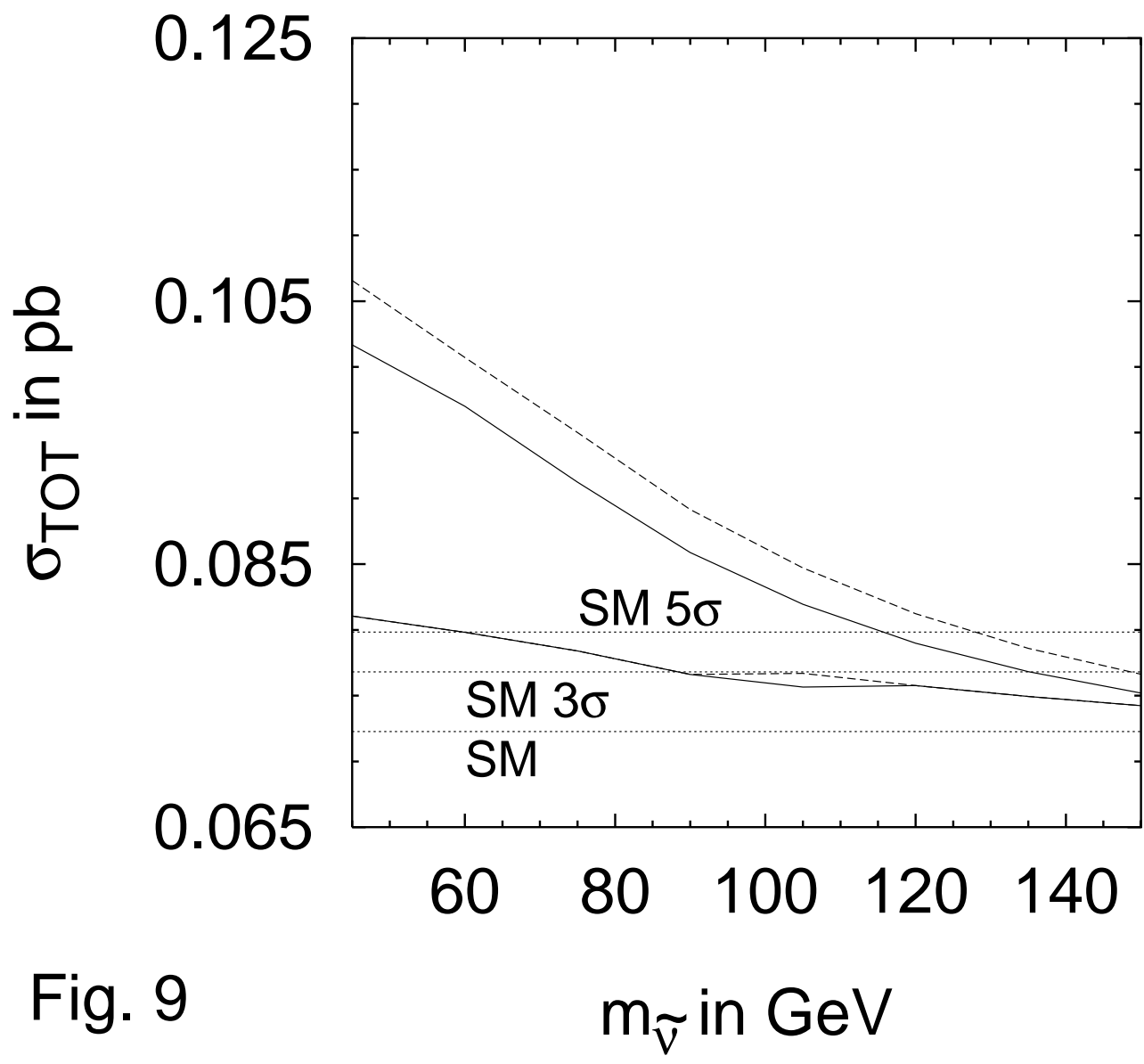


Fig. 9

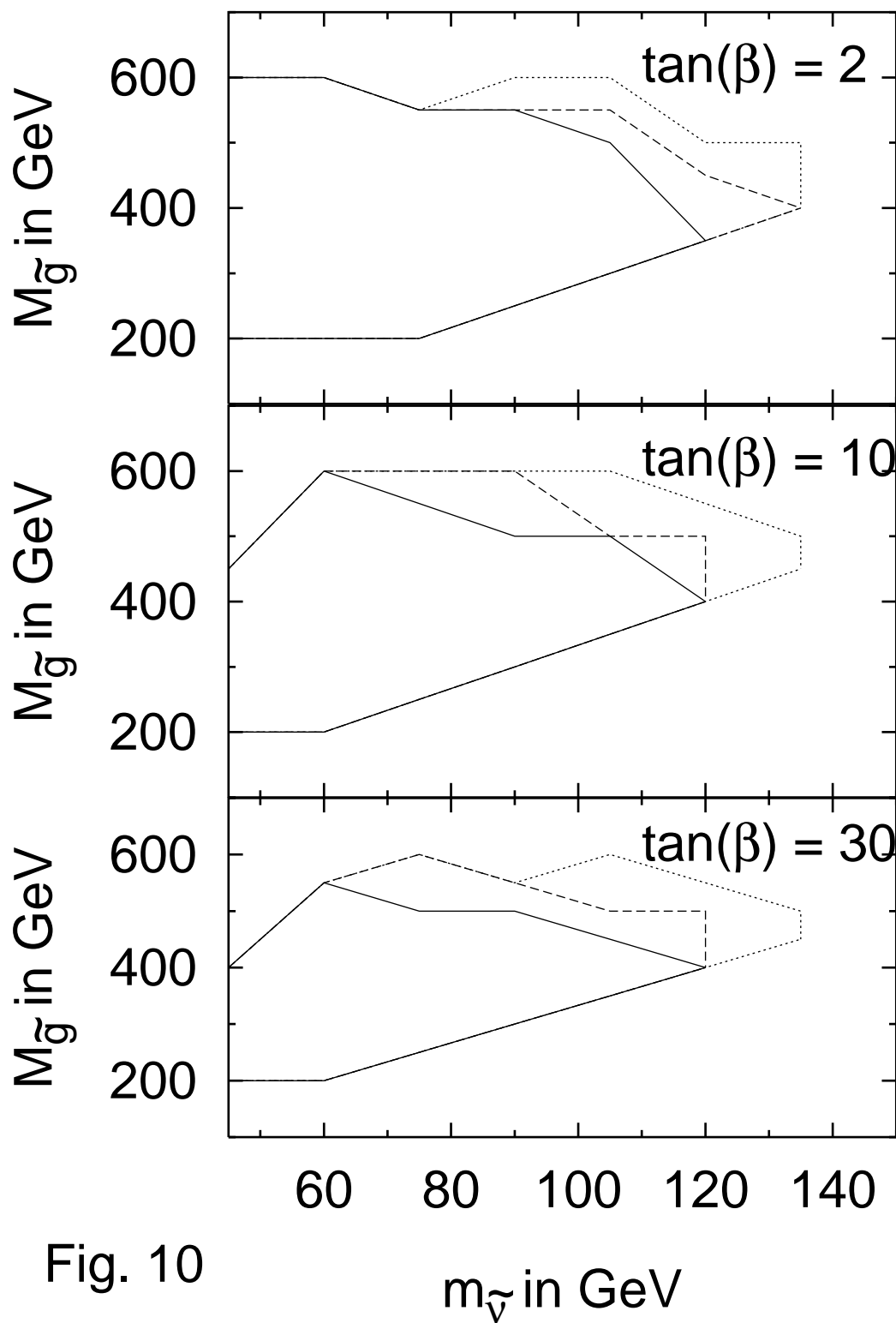


Fig. 10

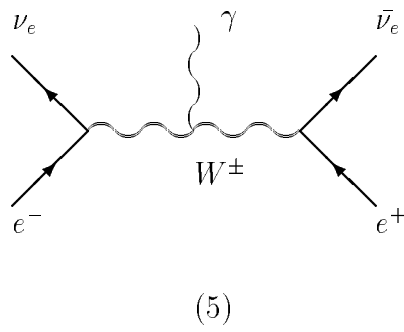
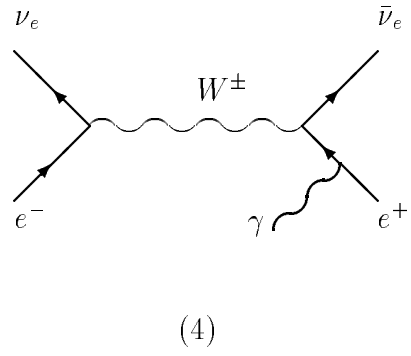
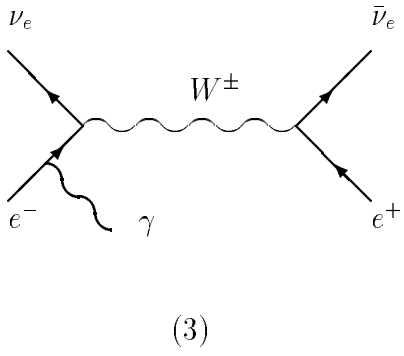
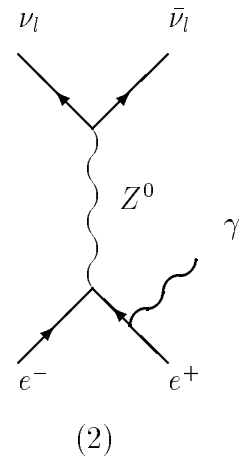
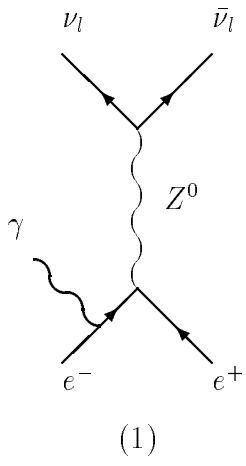


Fig. 11

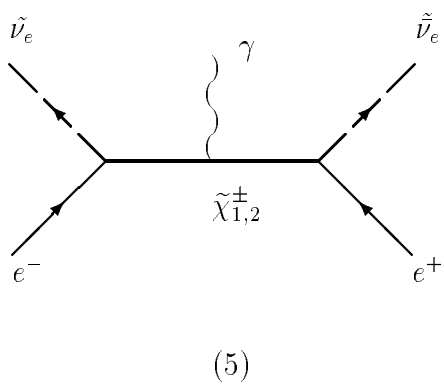
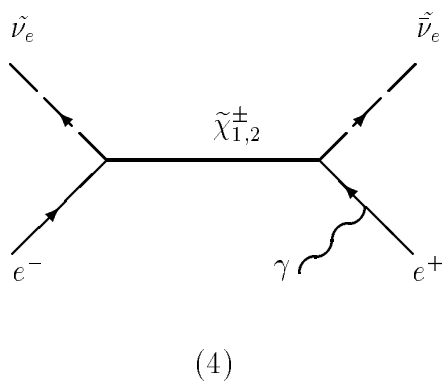
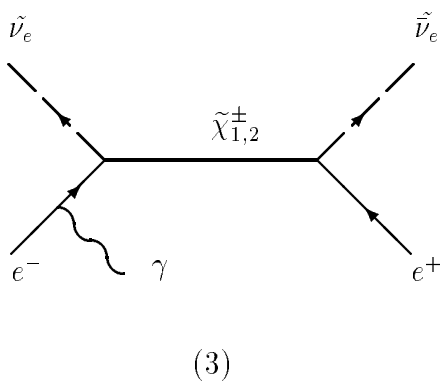
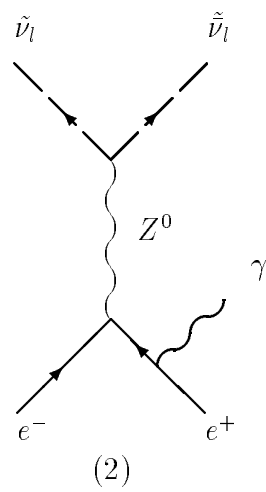
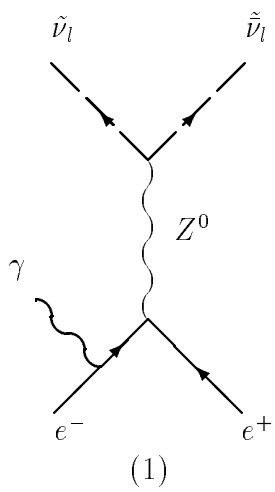
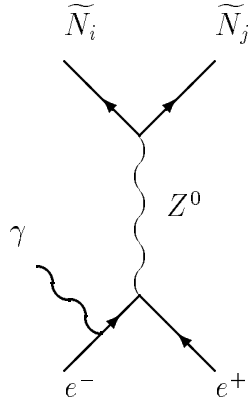
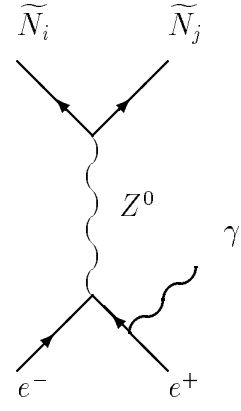


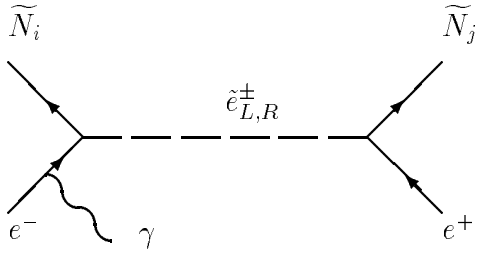
Fig. 12



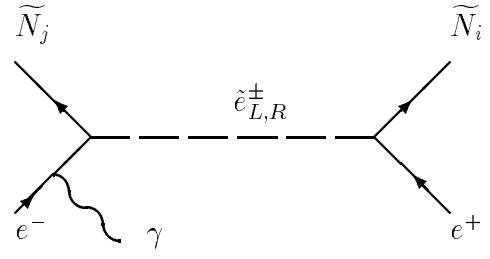
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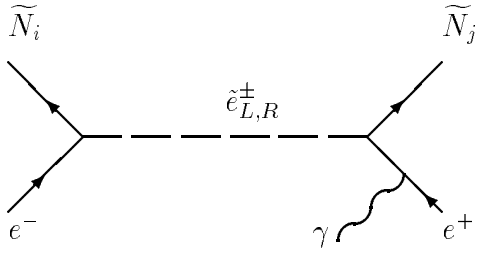
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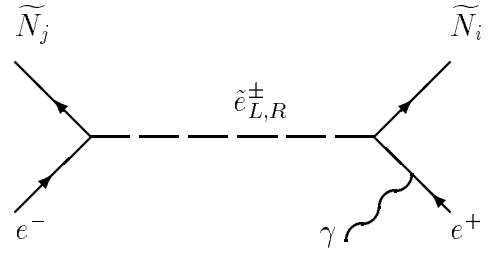
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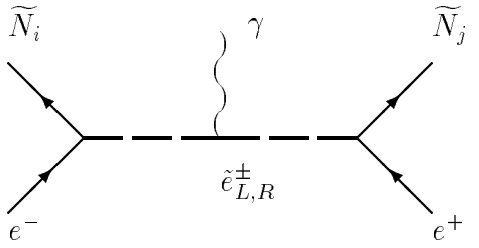
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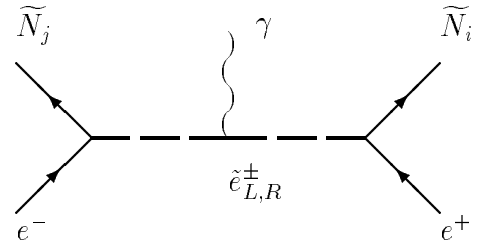
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(7)



(8)

Fig. 13